THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XVII MARCH 1903 NUMBER 2

THE "SOLAR CONSTANT" AND RELATED PROBLEMS.[1]

By S. P. Langley.

Physical astronomers, armed with new methods and perfected appliances, are helping us to a view of the progress of creation, from its beginning in the nebula, which must interest every student of nature. But however much our attention is aroused by the purely scientific aspect of such general studies, we must, it seems to me, consider, in the case I have now to present, *utility*, even before abstract interest. I refer to the study of the Sun, for though the most unformed nebula may hold the germs of future worlds, yet for us these possibilities are but interesting conjectures. For, as I have said elsewhere, I recognize that every nebula might be wiped out of the sky tonight without affecting the price of a laborer's dinner, while a small change in the solar radiation may conceivably cause the deaths of numberless men in an Indian famine.

From the foundation of the Smithsonian Astrophysical Observatory until now, I have therefore directed its work

[1]Read before the Washington meeting of the American Association for the Advancement of Science, December 30, 1902.

toward solar study, with a view to its probable utilities as well as to its purely scientific value, while still regarding this last as of high importance.

While the Sun, then, can be viewed merely as the nearest and most accessible star, yet it is here considered in a more important aspect to us, as the source of the radiation on which all human life depends.

1. *Early work at the Allegheny Observatory and on Mount Whitney.*—Some twenty years ago it was my good fortune to devise a successful means of measuring those minute quantities of heat on which depends an accurate determination of the possible variation in the enormous quantities which the Sun sends the Earth. This instrument, the bolometer, I used for nearly a decade at the Allegheny Observatory in the investigation of the Sun. It early appeared that many questions of the gravest importance, especially that of the so-called "solar constant," depended for their solution on the observer's stationing himself as high above our lower air as practicable. Accordingly, when through public and private channels the necessary means were at length provided, I conducted an expedition to Mount Whitney, in southern California. This expedition was provided with such essentials of a solar observatory as could be gathered and transported with the limited means at command, and was fortunate in including in its personnel the youthful Keeler, whose untimely end we all of us regret.

The observations secured at Mount Whitney confirmed my conclusion that the earlier values of the "solar constant" by Pouillet and others were too low. They had been obtained by the aid of the actinometer or the pyrheliometer—instruments which were made to father results that they were not strictly responsible for, through the use of wrong methods of extrapolation. It had been tacitly assumed that all solar rays are equally absorbed, or at least that all are similarly absorbed, in passing through successive layers of the Earth's atmosphere. Nothing could be more incorrect. Some large gaps are left in the spectrum where, for a range of wave-lengths as great as from the violet to the red, almost no energy remains after the solar beam

has passed through half the layer of air between the Sun and sea level, and I have shown that the absence of the study of individual rays—a study since made possible by the bolometer—leads to an altogether too small value of the solar constant.

It was, in short, shown by these observations that the solar radiations must be analyzed and their amount measured wave-length by wave-length, and coefficients of atmospheric transmission worked out for narrow spectral regions, before any trustworthy value could be assigned to the solar radiation outside the Earth's atmosphere. It is satisfactory to see how thoroughly this view, novel enough then, has now been accepted. Its results[1] gave values of from 3 to 3.5 calories for the solar constant, thus nearly doubling the classical value, 1.76 calories, of Pouillet. They brought no certainty of the fixity of this so-called "constant," which, however, indications now lead us to consider as possibly variable, and one whose changes are not improbably of the deepest practical concern to us. Where the most elaborate determinations of a quantity by competent observers thus differ by nearly 100 per cent., it is evident that, if there be considerable real variations in this quantity, they may exist without detection.

Were there no intervening terrestrial atmosphere, we might determine these solar variations, if they exist, by observations made here; but the difficulties of such a research, when conducted anywhere through the Earth's actually intervening and changing atmosphere, are manifest, and in our actual station perhaps insuperable. I shall proceed to give an example of the best means at our command for overcoming them—means admittedly insufficient here, but which we may hope to employ later in a more favorable locality, at an elevated station. This done, it will remain to consider an independent method which, by direct study of the Sun's atmospheric envelope, holds out some possibility of success in detecting variations in the Sun's radiation, if these exist, even in our actually unfavorable conditions.

[1] See *Professional Papers, Signal Service*, XV, "Report of the Mount Whitney Expedition."

2. *Improvements in apparatus.*—Since the institution of the Smithsonian Astrophysical Observatory, almost every year has witnessed improvements of the apparatus employed for making solar energy spectrum observations, such as are necessary in the problems relating to the solar constant. The story of all these improvements is quite too long to tell here, but can be found in Vol. I of the Observatory's *Annals*.

In consequence of these changes, which have now occupied me so many years, with final success, where two years of my personal assiduous labor formerly passed (at Allegheny) in getting an imperfect picture of the distribution of the energy in the solar spectrum, extending from the violet down to a wave-length of 2.5μ—this can now be obtained with ease in fifteen minutes of time, by an almost automatic process.

This stretch of spectrum contains almost all the energy which reaches sea level, and includes many regions of powerful water vapor absorption. It extends into a portion of the infra-red entirely unknown before the time of the Mount Whitney expedition, and which was, in fact, discovered in my observations on Mount Whitney itself.

You see superposed upon this chart (Plate VII) before you five of the energy spectrum curves to which I have just referred. The curves were taken at intervals of about one hour in two different afternoons. You will note that, while the five are generally similar, the lower pair shows greater absorption by water vapor in the great infra-red bands.

3. *New work of the Smithsonian Observatory.*—Armed now with apparatus capable of such results as I have just noted, and with the ability to gather in a quarter of an hour observations which could not have been obtained with equal accuracy in years of former work, the problem of the solar constant and the absorption of the solar radiation within the solar envelope, and within the Earth's atmosphere, has, after long intermission, now been taken up again at the Smithsonian Astrophysical Observatory, not with the expectation that a final solution can be reached, in its unfavorable local conditions, but with the hope that we may reach a useful approximate one, and become

familiar in practice here with methods to be carried out later, it is hoped, under more favorable conditions.

4. *The method of study.*—The work now commenced here is planned to include the following parts:

a) A study of the yearly variations of the selective absorption of the Earth's atmosphere by the aid of a long series of bolographs.

b) The production of bolographic energy spectrum curves at different altitudes of the Sun for the purpose of obtaining coefficients of atmospheric transmission at any desired wave-lengths.

c) Experiments to determine the loss experienced by the radiations in passing through the various optical parts of the bolographic apparatus.

d) The standardization of the bolographic apparatus by actinometer observations simultaneous with the bolographs.

e) The reduction of these several kinds of data to furnish coefficients of atmospheric transmission, coefficients of transmission of the apparatus, and ultimately means of getting values of the solar constant.

These five classes of studies refer mostly to absorptions within the Earth's atmosphere, but finally, and of the greatest importance to the immediate study of the variations of solar radiation, comes a study of the absorption of the solar envelope for any desired wave-lengths by means of bolographs taken at selected points on a large solar image.

5. *Selective absorption of water vapor.*—Continuing our consideration of the absorptions in the Earth's atmosphere, we observe that this chart (Plate VIII) gives a summary of eight months of bolographic spectral actinometry. The bolographs taken near noon of all clear days since February, 1902, have been reduced to a uniform scale of ordinates by a method which need not here be described, and their areas between wave-lengths 0.76μ and 2.0μ have been obtained. These areas are proportional to the total solar radiation between wave-lengths 0.76μ and 2μ which reaches the sea level.

In this infra-red portion occur the great water vapor absorption bands $\rho\sigma\tau$, ϕ, ψ, and Ω, but there is some reason to suppose

that the percentage absorption of water vapor is no greater here than in the visible spectrum as a whole.

The areas measured are plotted in the upper curve of Plate VIII. It will be seen how broken and variable the height is, but how the general slope indicates a considerable falling off in energy in the summer months. The reason of this appears in the next lower curves. Of these the dotted one gives the summation of the areas of those parts of the bolographs known to suffer greatly from water vapor absorption, and the full one gives the remainder not so much affected by it. You see how obviously our solar radiation is affected by the water vapor in the air. The difference between March and August amounts to about 20 per cent. It has been pointed out in Vol. I of the *Smithsonian Astrophysical Observatory Annals* that this water vapor absorption is annually periodic in its fluctuations, and is at a minimum in spring. A secondary minimum occurs in the autumn.

6. *The general absorption of the Earth's atmosphere.*— During the past year attempts have been made on each promising day to get a series of solar energy curves taken with different altitudes of the Sun, beginning in the morning and continuing every hour till late in the afternoon. Many of these series were stopped or rendered valueless by the appearance of clouds or haziness; for it is, indeed, surprising to find how little apparent change in the clearness of the air alters decidedly its transparency for some wave-lengths.

The method of reduction of the observations is as follows: A number of points on the bolographs are selected where there are no prominent bands of selective atmospheric absorption. At these points the heights are proportional to the deflections of the galvanometer and to the amounts of solar radiation of certain known wave-lengths which fall upon the bolometer. For each bolograph the time of passing these selected points is known, and therefore the altitude of the Sun.

We thus have all the data for fixing the mass of air traversed by the solar beam on several occasions, and the relative amounts of radiation of certain wave-lengths remaining after the passage through the air column. We assume now the validity of the

familiar formula of Bouguer, which seems, as applied to individual wave-lengths, to express well the results of these experiments.[1]

Such a procedure as I have indicated, applied to the work of six of the best days, has yielded the results which appear on this chart (Plate IX) before you. The six thin broken lines show the values of atmospheric transmission coefficients determined for the twelve different wave-lengths as indicated by the abscissæ of the chart. The heavy line represents the mean result of these six different days of observation. You will see how well the results generally agree. They were taken from different seasons of the year and very different air masses. The significant depression of the curve as it ends in the infra-red and also near the D lines I suppose to be due to a general absorption of water vapor in these spectral regions.

Another interesting feature of the work is that in the morning hours the transparency of the air generally increases rapidly and sometimes irregularly, while in the afternoon it continues longer of nearly uniform and maximum transparency. This result is shown by this chart (Plate X), in which the lines are plotted from the values of log d and $m\frac{B}{B_0}$ (see footnote, p. 95)

[1] If we suppose e to represent the amount of radiant energy per square centimeter of the given wave-length which reaches the Earth's surface; e_0 the amount prior to absorption by the air; a the proportion transmitted by a unit air mass, represented by vertical transmission through an air column of barometric pressure B_0, then for the given barometric pressure B and for the air mass m we have by Bouguer's formula:

$$e = e_0 a^{m\frac{B}{B_0}} .$$

But since the height of the bolograph d at the given wave-length is proportional to e, we may write, by introducing an instrumental constant k,

$$d = ke = ke_0 a^{m\frac{B}{B_0}} .$$

It is convenient for graphical computation to put this formula in logarithmic form as follows:

$$\log d = m\frac{B}{B_0}\log a + \log ke_0 .$$

The last member of this equation is supposedly constant, and hence we have here the equation of a straight line. Accordingly, if all the observed values of log d and $m\frac{B}{B_0}$ are plotted on any convenient scale, the quantity log a appears as the tangent of the inclination of the line determined by them.

for two different days at a wave-length of 0.51μ. As I have said, all the observed points should lie upon a straight line the tangent of whose inclination is the value of log *a*. It will be noted that the morning hours yield much steeper and more irregular lines than the afternoon hours, indicating the increase of transparency, which culminates shortly after noon.[1]

This general result of the preliminary experiments on the Earth's atmosphere may be of interest to meteorologists.

7. *The absorption of the apparatus and its standardization by the actinometer measures.*—Let us recall that before reaching the bolometer each ray of light, of whatever wave-length, is diminished in intensity, first by the absorption of the solar envelope, second by the absorption and diffuse reflection of the Earth's atmosphere, third by a similar loss at the siderostat mirror, and fourth within the spectroscopic train. These several losses are not the same for different wave-lengths, but all tend to increase as we pass from the infra-red into and through the visible spectrum.

The atmospheric transmission has just been mentioned, and we will now consider the losses within the train of apparatus. In my earlier work at Allegheny, I determined the reflection coefficients for silvered glass mirrors, and more recently other investigators have done the same. I do not therefore stop to give more recent results obtained here, as they are in substantial agreement with these others, but it may be well to say that, inasmuch as the reflecting power of the silvered siderostat mirror rapidly deteriorates in the visible spectrum, it is necessary to determine this quantity frequently in solar constant work.

Similarly, the three mirrors of the spectroscope deteriorate, and require that frequent determinations of the relative transmission of the spectro-bolometer for all wave-lengths should be made. I do not here detail the elaborate procedure adopted for this purpose.

Referring to Plate VII again, you see before you five solar

[1] It is at the same time to be noted that conditions of "good seeing" are usually associated with the morning hours.

spectrum energy curves. Take any one of these in illustration. Its ordinates represent intensities of radiation for all wave-lengths from 0.45μ to 2.5μ. The area bounded by this curve and the axis of abscissæ is therefore proportional to the total solar radiation between these wave-lengths which reaches the bolometer. If now all the ordinates should be increased by dividing their lengths by the corresponding transmission coefficients of the spectro-bolometer, we should obtain a new corrected energy curve. This curve may be extended by extrapolation to include the small amounts of solar radiation not included between wave-lengths 0.45μ and 2.5μ,[1] and its area will then be proportional to the total solar radiation in the beam reflected by the siderostat mirror.

8. *A provisional value of the solar constant.*—In the steps which follow, the necessary data are introduced to furnish a provisional value of the solar constant.

The energy curve is corrected for the loss at the siderostat mirror, and further, by the application of Bouguer's formula and the atmospheric transmission coefficients, the form of the energy curve outside the Earth's atmosphere is reached. Before this final step all the great terrestrial bands are smoothed over. Plate XI illustrates the several steps by means of which the bolograph is corrected to exhibit the distribution of the solar energy outside the Earth's atmosphere. Curve (1) is the bolograph itself. Curve (2) is corrected for relative absorption in the spectroscope. This curve is extended at either end to include what may reasonably be estimated as the solar radiation beyond its limits. The shaded area under curve (2) represents an actinometer reading of 1.36 calories per square centimeter per minute.

Curves (3) and (4) give the distribution of the solar radiation in the prismatic spectrum respectively at the Earth's surface and outside the atmosphere. The area under curve (4) is 1.87 times the area under curve (2), and we therefore find from this

[1] In a more exact determination of the solar constant these amounts would require careful measurement, but in this illustration only a rough extrapolation is attempted.

single determination that the solar constant is 2.54 calories per square centimeter per minute.

(The reader will understand that this is but a provisional value, given to illustrate the method, which, it is hoped, may be pursued later under more favorable local conditions. It is, indeed, unlikely that even by much greater labor we can obtain an accurate value of the constant from a station so near sea level as Washington, where the atmospheric absorption is so large and doubtful a factor. It will be remembered also that the writer has elsewhere shown that such values obtained near sea level are always of necessity too small.)

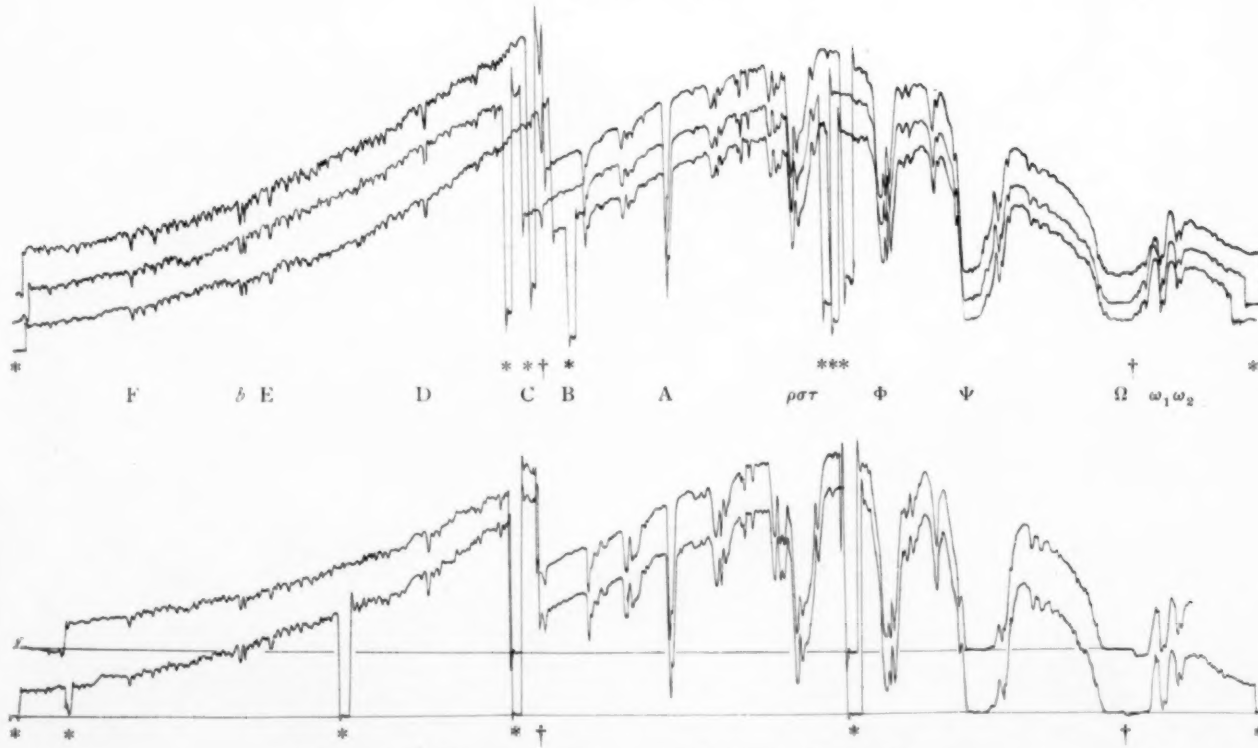
I cannot leave this part of the subject without acknowledging my constant obligation to my assistants, Mr. C. G. Abbot and Mr. F. E. Fowle, for aid in every part of the work.

We now reach the most important part of our study of the so-called solar constant, and of its possible variation.

9. *General absorption of the solar envelope.*—A chief cause of possible variability of the solar radiation is in the solar envelope itself. The absorption of this envelope may well be variable, as I have remarked in earlier articles,[1] in which I have indicated how far the surface temperature of our planet may depend upon it, and Halm has recently[2] put forth an ingenious theory to explain the Sun-spot cycle on this basis. I refer to this now only to point out that bolometric work upon the direct solar image and on the relative amounts of radiation of all wave-lengths, and from its various parts formerly carried on by me elsewhere, has been now taken up at the Smithsonian Observatory, and that a description of it will probably form the subject of a later memoir. Inasmuch as we receive heat and light through different thicknesses of the solar envelope at different parts of the disk, we have here, with the thermal study of faculæ, prominences, and spots, a means of studying solar absorption and radiation which, if continued (as I hope it may be) over a Sun-spot cycle, will determine our knowledge of whether the Sun's radiations to the Earth vary, as I am disposed to believe, from month to month and year to year; and which may, it seems

[1] *Am. Jour. Sci.*, **10**, 489, 1875. [2] *Nature*, February 13, 1902.

PLATE VII.

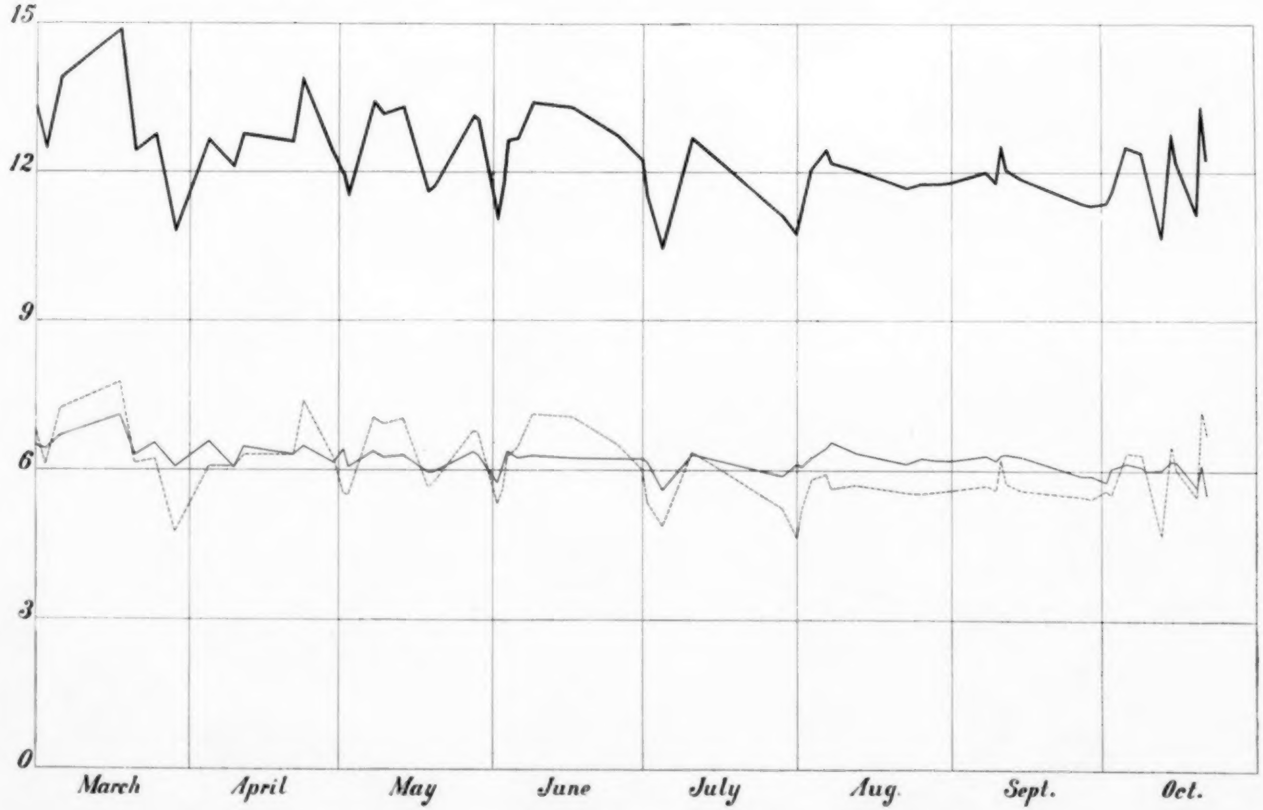


BOLOGRAPHIC ENERGY CURVES OF THE PRISMATIC SOLAR SPECTRUM.

Each curve made in fifteen minutes. The three upper curves taken on a day of moderate water vapor absorption; the two lower curves indicate much greater absorption. See bands $\rho\sigma\tau$, Φ, Ψ.

* Shutter closed at these points to give zero line. † Height of slit altered at these points.

PLATE VIII.

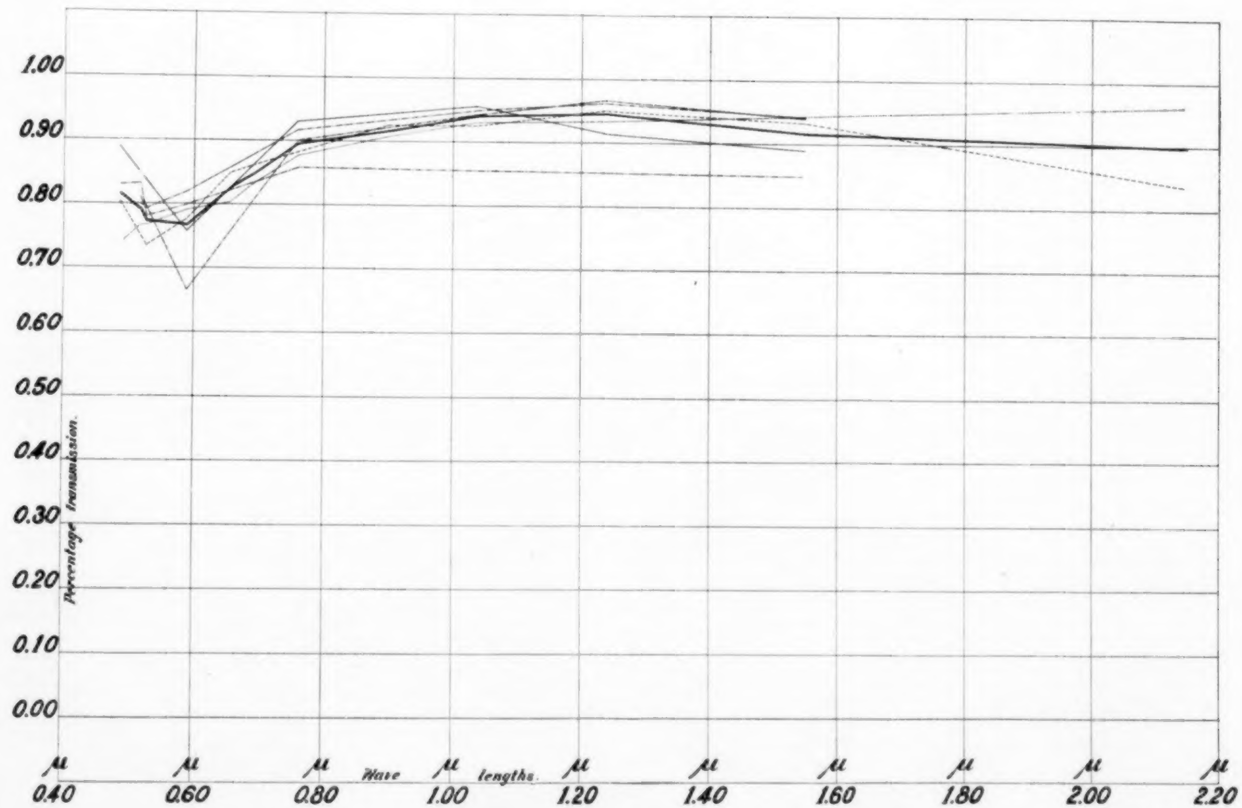


VARIATION OF SOLAR RADIATION CAUSED BY DIFFERENCES OF ATMOSPHERIC ABSORPTION.

(From Bolographic Studies of 1902.)

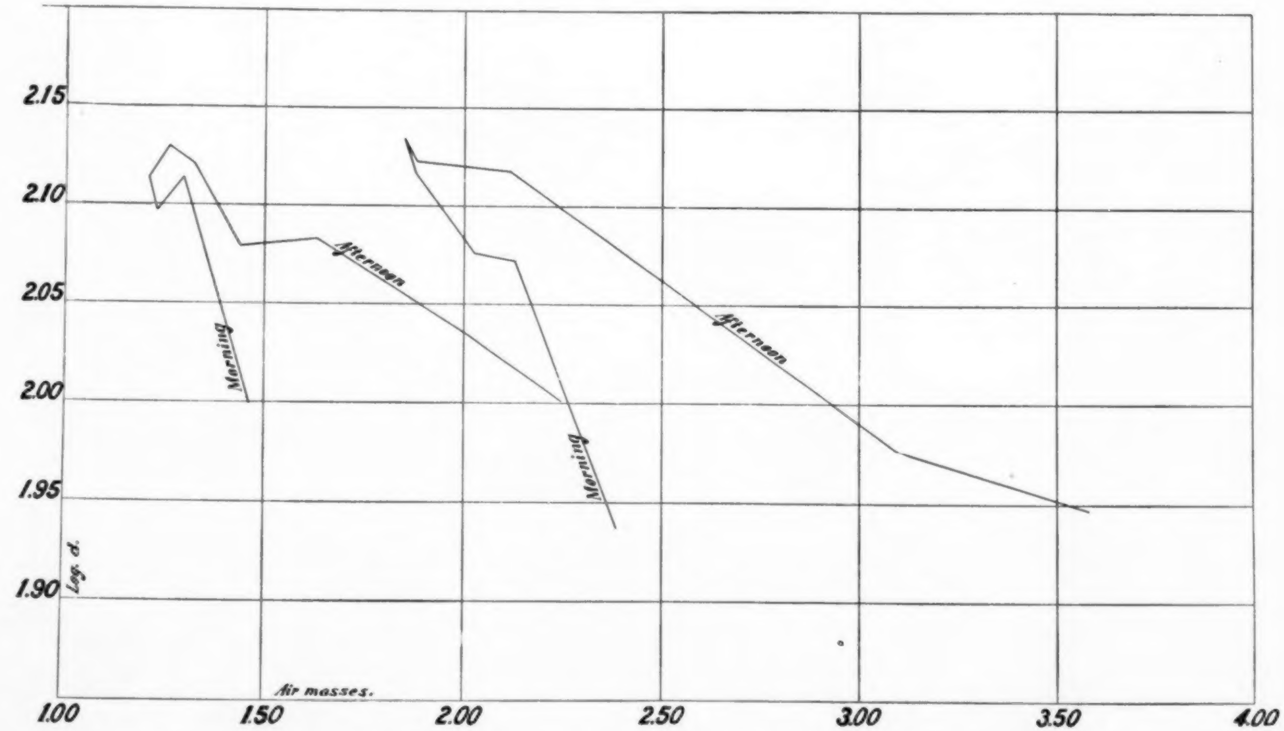
Upper curve represents total radiation between wave-lengths 0.76 μ and 2.0 μ.
Dotted curve represents portion of this affected by the great water absorption bands.
Lower full curve represents the remainder relatively unaffected by absorption of water vapor.

PLATE IX.



TRANSMISSION OF EARTH'S ATMOSPHERE AT WASHINGTON FOR VERTICAL AIR COLUMN AT 76 CM BAROMETRIC PRESSURE.

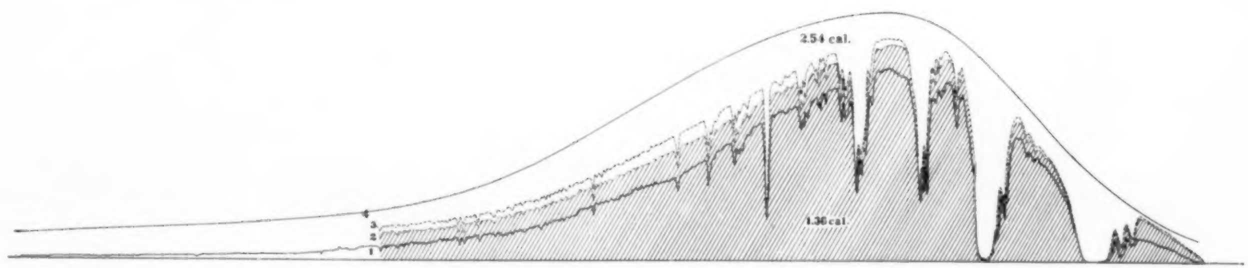
The heavy line is the mean of the results of six afternoons' bolographic observations.



VARIATION OF ATMOSPHERIC TRANSMISSION AT WASHINGTON FOR MORNING AND AFTERNOON HOURS.

The observations are for wave-length $0.51\,\mu$ and for two different good days. If the transparency of the atmosphere remained uniform, all the points plotted for each day should lie upon a straight line, the tangent of whose inclination is log *a* of Bouguer's formula.

PLATE XI.



BOLOGRAPHIC METHOD OF DETERMINING THE SOLAR RADIATION CONSTANT.

Curve 1 is the original bolograph.
Curve 2, same corrected for absorption of spectroscope, and representing by its shaded area a known actinometer reading.
Curve 3, same corrected for absorption of siderostat mirror.
Curve 4, same corrected for absorption of Earth's atmosphere.

increasingly probable, ultimately bring to light evidence of variation in the heat it sends to the Earth, which may not only interest the astronomer, but be of practical consequence to all men.

SUPPLEMENTARY.

Need of an elevated solar observatory.—The Earth's temperature and the life of its inhabitants, both animal and vegetable, depend on the solar radiation. Yet we confess that even at this late day we do not know, with any certainty, what the total amount of solar radiation is, whether it is constant or variable, or what effect upon terrestrial life and temperature a given change in it would produce. Our ignorance of these fundamental things is largely, though not wholly, due to the variability of our own atmosphere, which prevents us from studying that of the Sun.

The preceding observations are carried on here with extreme difficulty under the almost prohibitive local conditions for such work. We must at least attempt a determination of the solar radiation, and its possible variation under these conditions; although what is more urgently needed, as I believe, than any other desideratum of physical astronomy, is to establish an observatory, placed in some clear and elevated region and charged solely with problems relating to the possible variations of the heat the Earth receives from the Sun. I have referred to the former expedition to Mount Whitney for this purpose, but it is not a temporary expedition, but many years' occupation which is now in question. Now that great undertakings are the order of the day, let us hope that some way may open to reach the solution of a problem which so concerns the whole human race.

SMITHSONIAN INSTITUTION,
Washington, D. C., February 7, 1903.

ON THE OPTICAL CONDITIONS REQUIRED TO SECURE MAXIMUM ACCURACY OF MEASUREMENT IN THE USE OF THE TELESCOPE AND SPECTROSCOPE.

By F. L. O. Wadsworth.

(*Concluded from p. 19, Vol. 17, January 1903.*)

The effect of curvature of the prism faces or of varying optical density in the material of which they are made has already been investigated in part for plane wave-fronts (Vol. **16**, pp. 289–294). So far as this effect is individually considered it will be practically the same with spherical wave-fronts of small curvature as with plane wave-fronts, but it may happen that in the case of spherical wave-fronts the unsymmetrical aberration due to either or both of these causes may be in the same direction as that due to the prism train itself, or it may be in the opposite direction. In the first case the two effects are additive and the amount of permissible aberration due to each is correspondingly reduced, *i. e.*, for a given error of setting of the slit the amount of permissible temperature variation in the prism train is less than that indicated in (42),(44) and (45) by an amount depending on the sphericity of the incident wave-front. In the latter case the effects are to a certain degree compensatory, but, on account of the different form of distortion given to the wave-front by the two causes, the compensation can be exact only under certain conditions. Thus in the case of a single prism, which has also been investigated by Lord Rayleigh,[1] it is found that the aberration due to sphericity of the incident wave-front and the aberration due to curvature of the prism faces can destroy each other only when the curvature of the two faces is in opposite directions (*i. e.*, one face convex and the other face concave) and very nearly equal. In the case of varying optical density it is necessary to secure compensation to have at least two prisms in the train whose variations are in opposite directions.

[1] *Phil. Mag.*, **9**, 46–48.

From these general considerations it is evident without further investigation that, so far as the limiting aberration in the beam emerging from the spectroscope train is concerned, we cannot rely upon compensatory effects in enabling us to reduce the rigor of the conditions already imposed in (42) and (66), since any one or all of the effects involved are liable to change in amount, and even in sign with changes of temperature. The best that we can do in any case is to so adjust the focus of the collimator and the order of succession in the prism train, that

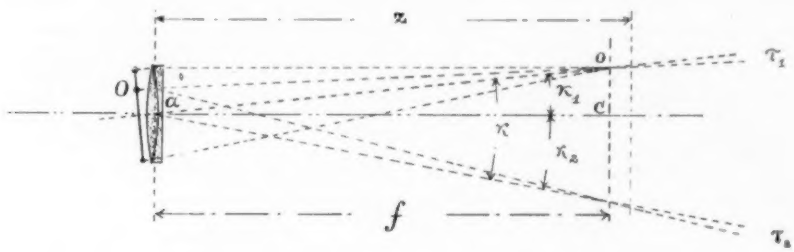


FIG. 11.

under what might be termed normal conditions of temperature and adjustment the wave-front issuing from the final prism face at minimum deviation is most nearly plane; or, in other words, to determine by one of the methods which will be mentioned later the accurate focal length of the collimator and prism train *combined*, not of the collimator alone as usually done, and then to set the slit as indicated on pp. 17 and 18.

When the incident wave-front is not symmetrical with reference to the axis of the instrument, the displacement of the image of a point or line due to a change in focus will depend on the distribution in intensity in the diffraction image in planes outside the principal focal plane. The detailed investigation of this problem in the case of a beam affected by unsymmetrical aberration or eccentrical incidence is a matter of considerable complexity. In the present case we may assume that for small distances df from the true focal plane the center of intensity of the diffraction pattern will lie on a line which passes through the central point, o, of the image at the principal focus, and the

point O, which marks what may be termed the mean center of illumination of the incident wave-front. Let the angle between this line oO, and the secondary optical axis, oa, *i. e.*, the angle aoO, be τ. Then if the angle oac be κ as before, it is evident that the distance ξ, of an image from the central point c of the plane in which it lies, will be

$$\xi_1 = f \sin \kappa_1 + (z - f) \sin (\kappa_1 - \tau_1) \ . \tag{95}$$

The separation of two images which lie at equal distances on opposite sides of the optical axis and having a total angular separation κ will be

$$\xi = 2\xi_1 = f \sin \kappa + (z - f) \left\{ \sin \left(\frac{\kappa}{2} - \tau_1 \right) + \sin \left(\frac{\kappa}{2} + \tau_2 \right) \right\} ,$$

or since κ and τ_1, τ_2 are all very small angles

$$\xi = z \{ \kappa - (\tau_1 - \tau_2) \} + f (\tau_1 - \tau_2) \tag{96}$$

and

$$\left. \begin{aligned} d\xi &= dz \{ \kappa - (\tau_1 - \tau_2) \} + df (\tau_1 - \tau_2) \\ &= \frac{dz}{f} (\xi - S) + \frac{df}{f} S \end{aligned} \right\} , \tag{97}$$

where S is, as before, the total linear separation of the mean centers of illumination of the incident wave-fronts.

Comparing (97) with (58) we see that in this latter case the change in the separation of the two images involves the consideration of the change in both z and f. It is necessary therefore to know the law of variation of f with t to the same degree of exactness as that with which we know the coefficients of linear expansion a, a', and a'' involved in a change in z.

Expressing the values of dz and df in the same form as before [(63) and (94)], and taking into account the expansions of the micrometer screw and photographic plate, etc., as in (59) and (60), we finally obtain in this case:

First, for direct micrometric or heliometric measurements at the focal plane $f \cong z + d\Omega$,

$$\Delta\xi = \kappa (z + d\Omega)(a - a') \Delta t + d\Omega [\kappa - (\tau_1 - \tau_2)] \\ - (\tau_1 - \tau_2)(z + d\Omega)(a - a''') \Delta t \ , \tag{98}$$

and second, for comparator measurements on photographic plates

$$\sum_{t}^{T}\xi = \kappa\,(z + d\Omega)\left[(a - a')\,\Delta t + (a' - a'')\,\Delta' t\right] \\ + d\Omega\left[\kappa - (\tau_1 - \tau_2)\right] - (\tau_1 - \tau_2)\,(z + d\Omega)\,(a - a''')\,\Delta t \ . \qquad (99)$$

When $\tau_1 = \tau_2$, *i. e.*, when the mean centers of illumination S_1 and S_2 are incident at the same point on the objective, the measured separation between any two images is exactly the same in the case of eccentrical incidence as in the case of centrical incidence (64) and (65), previously considered. From this it follows that any unsymmetrical diaphragming or unsymmetrical absorption (the effect of which is the same) is without effect on $\Delta\xi$, provided only that the wave-fronts in each case fill the whole aperture. The position of each individual image, however, will be shifted by an amount indicated by differentiating (95), and the application of this principle forms the basis of the ingenious methods that have been described by Cornu,[1] Newall,[2] Hartmann,[3] and others for determining the exact focal length of a telescope.

If the focusing were always exact we would have

$$\begin{aligned} d\Omega &= (a - a''')\,f\Delta t \\ &\cong (a - a''')\,(z + d\Omega)\,\Delta t \ , \end{aligned} \qquad (100)$$

and under such circumstances (98) and (99) would again reduce to forms identical with (64) and (65). Exact focusing, in the sense in which the term is used in metrological work, is, however, never possible by the ordinary standards of definition; hence it is better to consider these terms separately.

The quantities a, a', a'', and a''', which appear in the first and last terms of (98) and (99) are all small and may be accurately determined once for all. Hence since $d\,\Omega$ is always small compared to z, and κ is known from the measurements themselves, we can, as we have already shown, determine the value of the first term of these corrections with all requisite accuracy if we know the value of Δt within 3° ($\pm 1\overset{\circ}{.}6$). In order to do the same for the other two terms we must first of all determine the values of the angles τ_1 and τ_2.

[1] *Ann. de l'École Normale*, (2) **9**, 21.

[2] *M. N.*, **57**, 572, 1897.

[3] ASTROPHYSICAL JOURNAL, **12**, 37, 1900.

The first and most interesting case to be considered is that of the prism spectrograph. Here the separation of the axial pencils of the two extreme wave-fronts is produced by the dispersion of the spectroscope train. The amount of this separation has already been calculated [see equation (34)].

If there were no absorption in the prism train, and the prisms and view telescope had apertures sufficiently large to transmit the entire beam of the two extreme lateral pencils, the separation S given by (34) would be that required in (97). Owing to the unsymmetrical absorption, however, the mean center of illumination O will not be on the axis of the wave-front, but will be displaced toward the refracting edge of the prism by an amount proportional to the coefficient of absorption. With the same notation as that already employed in (8) and (9) we have for the total quantity of light transmitted

$$\begin{aligned} A &= \int_{-\frac{b}{2}}^{+\frac{b}{2}} i_0 e^{-Bx}\, dx \\ &= \frac{i_0}{B}\left[e^{\frac{Bb}{2}} - e^{-\frac{Bb}{2}} \right] . \end{aligned} \tag{101}$$

Hence the mean center of illumination O will lie at a point S_0 such that

$$i_0 \int_{-S_0}^{\frac{b}{2}} e^{-Bx}\, dx = \frac{1}{2} A \ , \tag{102}$$

which by comparison with (101) gives at once

$$S_0 = -\frac{1}{B} \text{nap log} \frac{e^{\frac{Bb}{2}} + e^{-\frac{Bb}{2}}}{2} \ . \tag{103}$$

For $B = \frac{1.386}{b}$, as previously assumed,

$$S_0 \cong -\frac{1}{6} b \ , \tag{104}$$

and therefore

$$\tau_1 = \frac{S_0}{f} = -\frac{1}{3} \beta \ ,$$

i. e., the center of illumination is displaced one-sixth the entire diameter of the wave-front toward the refracting edge of the

prism train, and the angle τ_1 for the wave-front passing at minimum deviation is one-third the semi-angular aperture of the view telescope.

In order to avoid displacement of the central image (for which $\kappa = 0$) with change of focus, the center of the objective must coincide with the mean center of illumination O; *i. e.*, the principal axis of the view telescope should not lie on the axial line of the ray transmitted at minimum deviation but should be shifted toward the refracting edge of the prism train by the amount indicated in (103). This is a point generally overlooked in the design of spectrographs.

In the case of lateral pencils the value of the middle term representing the effect of the mechanical change in focus will be less or greater according as the signs of τ_1 and τ_2 are the same or opposite, and according as the sign of their differences is the same or opposite to the sign of κ. If τ_1 has the same sign as κ we may make the term $\kappa-(\tau_1-\tau_2)$ zero by properly controlling S_1 and S_2, the points of incidence of the centers of illumination of the lateral wave-fronts. In the case of the spectrograph this can be accomplished by so proportioning f, the focal length of the view telescope, and L, the total length of path through the prism train, that the value of S in (97) and (34) is identical with the value of ξ, the separation of the two images at the focal plane. This gives us, from (34) and (35),

$$f(\tau_1-\tau_2) = f\kappa = \frac{d\theta}{d\lambda}\left[\frac{1}{2}\left\{L_1 + 3L_2 + 5L_3 + \cdots\cdots (2N-1)L_N\right\} + n\left\{L_1 + 2L_2 + 3L_3 + \cdots\cdots N\frac{L_N}{2}\right\}\right], \tag{105}$$

or since the total angular displacement κ is $2N$ times the displacement $\frac{d\theta}{d\lambda}$ for one refraction, we have for three prisms

$$f = \frac{1}{12}\left[L_1 + 3L_2 + 5L_3\right] + \frac{1}{6}n\left[L_1 + 2L_2 + 3\frac{L_3}{2}\right]. \tag{106}$$

For the Bruce spectrograph already considered

$$f = 10.08 + 16.42 = 26.5 \text{ cm},$$

instead of 44.9 cm and 60.7 cm, as adopted for camera A and camera B.

Since (34) is deduced on the assumption that the prisms and objective of the view telescope are placed as close together as possible, (105) and (106) express the minimum value of f that will satisfy the above condition for the elimination of the term $\kappa-(\tau_1-\tau_2)$. We can, however, satisfy this condition for any *larger* value of f by separating the prisms or by withdrawing the camera objective so that the total path L through the spectroscope train is correspondingly increased. It is so easy to do this in the original design of the instrument, and the advantage resulting from it in the entire elimination of the effect of errors in the scale reading Ω is so great, that it is singular that this particular point of design seems to have been previously overlooked.

We may accomplish the same result in the case of the plane-grating spectrometer by making the distance from the grating to the objective of the view telescope equal to the focal length of the latter. In this case as before the objective of the view telescope must be increased in size sufficiently to receive the entire cross-section of the lateral beams. In this latter case the use for which the instrument is intended, as well as the form of view telescope adopted, must be considered before we can determine in any case whether it is worth while to introduce this modification of the usual design. With reflecting view telescopes the condition is very easily and almost necessarily satisfied, and this is another advantage of this form of grating spectroscope.[1]

In the case of the heliometer we may satisfy the same condition of zero displacement with change in the position of the eyepiece by proper diaphragming. With a full semi-circular aperture the total quantity of light in the transmitted wave-front is proportional to $\frac{1}{2}\pi r^2$. If we diaphragm one edge of the aperture by an amount mR the amount of light transmitted will be

$$\left.\begin{aligned} A &= \int_{-R}^{R(1-m)} \sqrt{R^2-x^2}\,dx \\ &= \frac{R^2}{2}\left[(1-m)\sqrt{2m-m^2}+\sin^{-1}(1-m)+\frac{1}{2}\pi\right] \end{aligned}\right\} \quad . \quad (107)$$

[1] *Phil. Mag.*, **38**, 137; ASTROPHYSICAL JOURNAL, **1**, 232, etc.

The mean center of illumination of the diaphragmed aperture lies at a point S_0 such that $S_0 = R(1-n)$ and

$$\int_{-R}^{-R(1-n)} \sqrt{R^2 - x^2}\, dx = \frac{1}{2} A \ ,$$

and therefore

$$\left.\begin{aligned} &2\ [(n-1)\sqrt{2n-n^2} + \sin^{-1}(n-1)] + \frac{\pi}{2} \\ &= (1-m)\sqrt{2m-m^2} + \sin^{-1}(1-m) = \frac{2}{R^2}A - \frac{\pi}{2} \end{aligned}\right\} = A_m \ . \qquad (108)$$

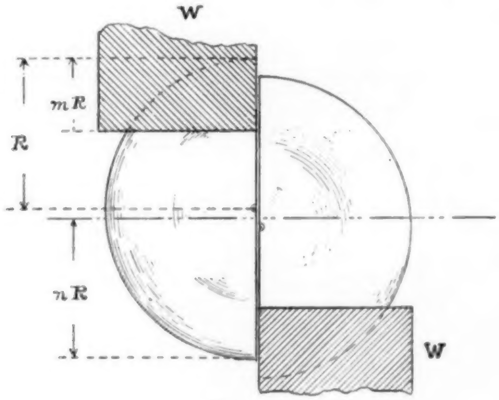


FIG. 12.

Let $A_0 = \frac{1}{2}\pi R^2$ be the amount of light transmitted when there is no diaphragm. Then we have from (107) and (108)

$$\frac{A}{A_0} = \frac{A_m}{\pi} + \frac{1}{2} \ . \qquad (109)$$

Equation (108) serves to determine m, the amount of diaphragming required for any given value of n, and (109) gives the ratio between the illumination secured with this diaphragm and the illumination with full (half) aperture. To determine n we have also in this case

$$S_0 = f\tau_1 = f\frac{\kappa}{2} = R(1-n) \ ,$$

whence

$$n = 1 - \frac{\kappa}{2\beta} \ . \qquad (110)$$

Assume as before that for a 16 cm heliometer $\beta \cong .03$. Then for two objects separated by an angular interval of 0°.5, $\frac{\kappa}{2} = 0.00436$ and $n \cong 0.855$, *i. e.*, the center of illumination of the incident wave-front should fall upon each half of the divided lens 0.145 of the radius from the center. This is accomplished by giving to m a value obtained by solving (108) for $n=0.855$. We thus obtain

$$m = 0.475 \; ,$$

i. e., a little less than one-fourth of each half of the objective should be covered, as in Fig. 12, the screens W W being placed so as to shut out in each case the light from the edge which is on the opposite side of the optical axis from the image formed by that half.

By expressing κ as a fractional part of β we can determine the general relation between κ and m for any heliometer. This has been done for values of $\frac{\kappa}{\beta}$ varying by tenths from 0 to 1 and by 0.2 from 1 to 2. The results for A_m, $\frac{A}{A_0}$, and m are given in Table IV and plotted in Fig. 13.

With this relation it is easy to devise a mechanical arrange-

TABLE IV.

$\frac{\kappa}{\beta}$	n	A_m	$\frac{A}{A_0}$	m
0	1.00	1.5708	1.00	0
0.1	0.95	1.3706	0.9363	0.230
0.2	.90	1.1726	.8732	.369
0.3	.85	0.9738	.8099	.490
0.4	.80	.7774	.7473	.600
0.5	.75	.5808	.6849	.705
0.6	.70	.3876	.6233	.805
0.7	.65	.1994	.5634	.900
0.8	.60	.0144	.5046	.988
0.808	.596	.0000	.5000	1.000
0.9	.55	− .1664	.4471	1.085
1.0	.50	− .3422	.3911	1.172
1.2	.40	− .6770	.2845	1.345
1.4	.30	− .9784	.1886	1.512
1.6	.20	−1.2424	.1047	1.678
1.8	.10	−1.4542	.0372	1.840
2.0	.00	−1.5708	.0000	2.000

ment which will automatically move the screens *WW* of Fig. 12 by the required amount when the two halves of the object-glass are adjusted to bring two images separated by the angular interval κ into coincidence. The great advantage of such an arrangement in the case of the heliometer is that when it is adopted the eyepiece may be moved in or out at will to suit the convenience and personality of the observer without thereby introducing any differences or errors in the settings of the instrument itself. The only drawback is that the light is considerably reduced for objects having considerable separation.

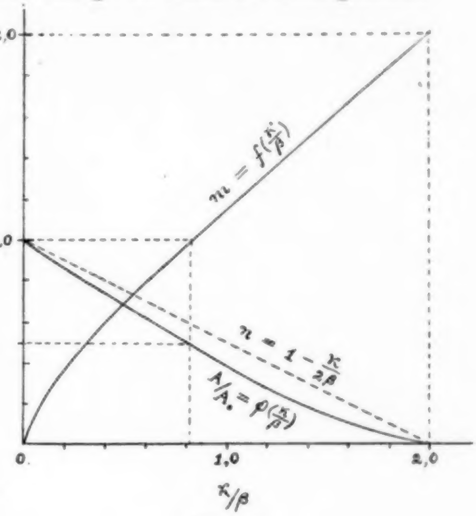


FIG. 13.

In this case, as in the preceding, the values of the angles τ_1 and τ_2, which appear in the third terms of (98) and (99), are determined by the relations

$$(\tau_1 - \tau_2)f = f(\kappa) \ ,$$

and these terms reduce to the form

$$\kappa f(a - a''')\,\Delta t \ , \qquad (111)$$

in which all the quantities are known or may be measured, and which, like the first terms of these equations, may be computed with all necessary accuracy when we know Δt for ordinary object-glasses to within about 3°. Temperatures of the object-glass and telescope-tube may be easily observed with this degree of accuracy, but we may by the special optical construction already mentioned on page 19 eliminate this term entirely by making $a - a''' = 0$.

B (3). It would be impossible and, in fact, quite unnecessary to enter here upon a general discussion of the methods of optical measurement best suited to individual cases. We shall endeavor

to point out only some general principles of working which the previous investigations have shown that it is necessary to regard if we aim at the highest attainable degree of accuracy in our work, and to investigate in this connection one or two individual sources of error in the use of particular instruments which seem either to have escaped previous attention, or at least to have been considered less carefully than their importance demands.

In general, the causes will produce the maximum effects in the optical distortion and displacement of images, and the ones therefore which we must most carefully guard against are those dealt with in sections *A* (2), *A* (3), *B* (1), and *B* (2).

In order to eliminate, or minimize as far as possible, the effects of asymmetrical, spherical, or chromatic aberration, A (2) and A (3), we must employ instruments and methods of work planned with special reference to *avoiding* the following sources of error: (1) the use of too large fields or of too great angles of incidence, particularly when using short focus mirrors or gratings at the principal focus; (2) the use of methods of comparison or of forms of instruments and of optical construction which necessitates that the different images whose relative positions are to be measured are formed by different portions of the surfaces of the optical train; (3) variations in temperature conditions during the interval of measurement or of record; (4) the effect of small chromatic dispersions, particularly when the spectral curve of radiation is different for the different sources; (5) the effect of varying chromatic absorption and unsymmetrical broadening of the spectral lines when the chromatic dispersion is large.

In the case of errors of Class B we must especially guard against (6) the use of too small resolving powers, and (7) the continued effect of temperature variations and asymmetrical illuminations.

Of these various effects those which have heretofore received most attention are (3) and (7). Temperature variations are, indeed, our greatest source of trouble and error in the great majority of physical measurements of all kinds. It appears, however, from the results of the investigations on this point

(pp. 292–4), that these effects are of even larger order of magnitude than is ordinarily assumed, and that in spectroscopic work particularly we must guard against them with the greatest care. As was pointed out on pp. 278, 279 and 292–4, the best method of eliminating residual effects, after reducing the temperature variations to the smallest possible range, is to measure the separation of the images and determine the so-called constants of the instrument, *i. e.*, focal length, pitch of measuring screw or scale, etc., simultaneously with each observation, instead of relying on values determined once for all at considerable intervals, as is usually done.

This method of simultaneous record is carried out most completely at present in determining star places by the chart photographs of the Astrophotographic Survey, in which the constants of reduction for each plate are determined from the photographic record on the plate itself. The next best example is that of the spectrographic method of determining absolute wavelengths and motions in the line of sight by the use of comparison spectra. The only criticism to the present practice of the method in the latter case is that the record of the comparison spectrum and of the spectrum to be measured is not simultaneous, and in the case of star spectra, in which an interval of sometimes two hours intervenes between the records, the residual errors of intermediate temperature variations, small though they may be, are in certain cases quite sensible. It would be much better, as already pointed out, if the exposure on the comparison spectra were made continuously instead of intermittently at the beginning and end of the exposure, and this could be arranged without any great difficulty.

It must be farther noted, however, that this method of simultaneous record and individual reduction of each set of measures, although it may eliminate almost completely the effect of small temperature variations, will not eliminate the effects of (1), of (2), of (4), of (5), nor of the latter part of (7), unless, indeed, the conditions of observation are precisely the same in all of these respects for both the objects upon which the direct measurements are made and the objects upon which the comparison meas-

urements for the "constants" of reduction are executed. It is physically impossible always to fulfill this requirement in the case of (4) and (5), and frequently very difficult in the case of (1), (2), and (6) because of the different parts of the field in which the two sets of images (direct and comparison) are situated, or because of necessary limitations in the size and power of the observing instrument.

For these reasons it is unsafe, if we aim at the highest degree of accuracy in optical measurements, to neglect the causes of optical displacement considered above, or to assume beforehand that their effects are vanishingly small. In this connection it is necessary to insist again on the point already made, that the ordinary tests and standards of optical definition are not sufficiently rigorous to serve as criteria in determining questions relating to the limiting accuracy of measurement. Nor are we safe in considering our results free from constant errors which may arise from instrumental causes, simply because they agree consistently among themselves, or are checked by occasional measurements of known quantities. Our only safe way of procedure would seem to be to investigate in each individual case, as we have already done in the general one, the *maximum individual* effects produced by possible disturbances of theoretical conditions, and either to reduce each of these maximum individual effects (unless there may be two or more which are *always necessarily* compensatory) to a quantity less than ϵ, the limiting metrological power; or to determine the magnitude of each effect to the same degree of exactness, in order that it may be taken care of as a known correction. In such individual cases there may exist possible causes of disturbance of a special character not included in the general classes of errors already discussed. As stated at the beginning of this section, it is unnecessary and undesirable to take up in detail a complete discussion of any number of such cases.

There is, however, one individual case in which an error of measurement due to a cause not yet considered is very likely to be introduced, and which on account of its importance in one of the leading fields of astrophysical research we will consider at

length. This is the case of the slit spectroscope or spectrograph as used for determinations of velocity of motions in the line of sight, or more generally of absolute wave-lengths. Here, in order either to secure greater intensity of spectrum, or to localize determinations with reference to different parts of the sources under examination, images of both the latter and of the comparison sources are first formed on the slit of the instrument, and we measure the relative positions, not of the spectral images of the sources themselves, as they would be formed by an objective prism or grating, but the relative positions of the spectral images of the slit as illuminated by the light concentrated upon it by the image-forming objective. The distinction is an important one, for, as we have seen in the case of A (3), any asymmetrical distribution in intensity in the source of radiation will result in an asymmetrical distribution in intensity in the image, and a consequent displacement of its center of maximum brightness. Hence if the slit is *not* uniformly illuminated across its entire width, its spectral image corresponding to any particular wave-length in the light falling upon it will be displaced from the position it would occupy under conditions of uniform illumination, no matter what the conditions in the source itself may be. The asymmetrical illumination of the slit may arise from (1) asymmetrical distribution of intensity in the image of the source itself on the slit, due either to a real asymmetry of radiation or to asymmetrical distortion or aberration of the image-forming objective; (2) displacements of the center of symmetry of the image with reference to the center of the slit. As the effect on the measurements will be the same in both cases we may consider them jointly.[1]

The general expression for the distribution in intensity in the

[1] This effect is entirely different in nature from that produced by lack of uniform illumination of the collimator objective, which has already been considered in *B* (2). So far as I have been able to ascertain, the effect of asymmetrical illumination of the slit itself it had escaped attention previous to the time when the preliminary results of this paper were informally announced at the Astrophysical Conference at the Yerkes Observatory in 1897. It was then expected that these results would be at once investigated in detail by others more directly interested than the writer in line of sight work, but their bearing on this line of research seems to have been overlooked or not fully realized by those present at that time.

spectral image of a slit *uniformly* illuminated across its entire width by radiation from any individual spectral line has already been given [equation (52)], and this expression has been evaluated for a number of special cases.[1] If the illumination across the width σ is not uniform, we must add another factor, $f(\xi)$, to express this variation in intensity, which gives us in (52).

$$I_{,,,} = A'' \int_{-\frac{\sigma}{2}}^{+\frac{\sigma}{2}} f(\xi)\, \psi_1 \{\xi - \psi_1\, a_0 a\}\, d\xi \ . \tag{112}$$

As in the consideration of a uniformly illuminated slit, we will first examine the case in which the light on the slit is absolutely monochromatic and the spectroscope itself is free from aberration. Then at the focal plane of the instrument the function ψ_1 becomes simply

$$\frac{\sin^2 \frac{\pi a}{a_0}}{\left(\frac{\pi a}{a_0}\right)^2},$$

and we have for $I_{,,,}$

$$I_{,,,} = A'' \int_{-\frac{\sigma}{2}}^{+\frac{\sigma}{2}} f(\xi)\, \frac{\sin^2 \frac{\pi}{a_0}(a - \xi)}{\frac{\pi}{a_0}(a - \xi)}\, d\xi\ , \tag{113}$$

which is the same form as that expressing the distribution in intensity in the image of an individual spectral line [equation (51)], for an infinitely narrow slit.

It is evident that (113) like (51) will be symmetrical or asymmetrical according to the form of the function $f(\xi)$. We will examine a few cases of particular interest in detail.

First, suppose

$$f(\xi) = A + B\xi\ . \tag{114}$$

This represents a uniformly progressive increase in the intensity of illumination across the entire width of the slit, such as is graphically illustrated in Fig. 14. Taking the origin of co-ordinates at the point corresponding to the geometrical image of

[1] *Loc. cit.*

one edge of the slit (instead of the center as before), the integral (113) becomes

$$I_{'''} = A'' \int_0^{\sigma} (A + B\xi) \frac{\sin^2 \frac{\pi}{a_0} \left\{ a - \left(\xi + \frac{\sigma}{2} \right) \right\}}{\left[\frac{\pi}{a_0} \left\{ a - \left(\xi + \frac{\sigma}{2} \right) \right\} \right]^2} d\xi \,. \qquad (115)$$

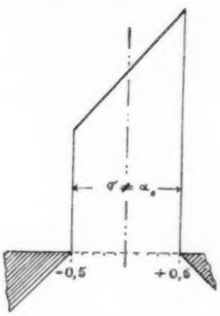


FIG. 14.

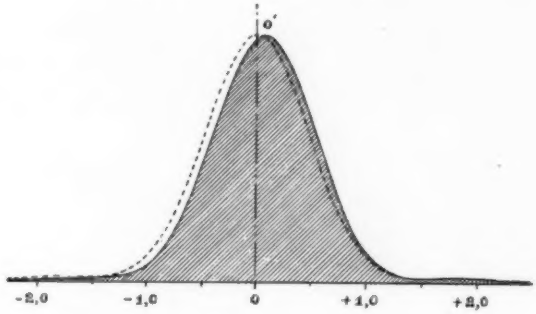


FIG. 15.

I have evaluated this integral by mechanical quadrature for the special case $A = \frac{1}{2}$, $B = \frac{1}{2a_0}$, and $\sigma = a_0$ (*i. e.*, the case in which the angular width of the slit is equal to the resolving power of the view telescope), and the intensity of illumination at one edge, $\xi = 0$, is one-half that at the other, $\xi = a_0 = \sigma$. The results to three places of decimals are tabulated in terms of fractional parts of $\frac{a}{a_0}$ in Table V and plotted in Fig. 15. In both the table and the figure the origin of co-ordinates has been shifted back to the center of the geometrical image.

An examination of these results shows that the position of maximum brightness in the spectral image of the slit is displaced in this case about $0.08\,a_0$ from the center of the geometrical image. This is just a little more than the limiting accuracy ϵ of the instrument, *i. e.*, a uniform variation of 50 per cent. in intensity of illumination from edge to edge of the slit will displace the center of the physical spectral image by an amount somewhat exceeding the limiting accuracy of measurement.

TABLE V.

$\frac{a}{a_0}$	$I_{...}=f(\xi)$	$\frac{a}{a_0}$	$I_{...}=f(\xi)$	$\frac{a}{a_0}$	$I_{...}=f(\xi)$
−2.45	0.011	−0.75	0.235	+0.95	0.148
−2.35	.011	−.65	.317	+1.05	.090
−2.25	.012	−.55	.429	+1.15	.054
−2.15	.014	−.45	.552	+1.25	.036
−2.05	.015	−.35	.672	+1.35	.030
−1.95	.018	−.25	.798	+1.45	.030
−1.85	.020	−.15	.898	+1.55	.032
−1.75	.023	−.05	.968	+1.65	.033
−1.65	.026	+.05	1.000	+1.75	.031
−1.55	.028	+.15	0.987	+1.85	.027
−1.45	.030	+.25	.933	+1.95	.022
−1.35	.032	+.35	.844	+2.05	.017
−1.25	.038	+.45	.727	+2.15	.013
−1.15	.048	+.55	.595	+2.25	.011
−1.05	.068	+.65	.462	+2.35	.010
−0.95	.102	+.75	.337	+2.45	.010
−0.85	.153	+.85	.231	......	....

Second, suppose

$$f(\xi) = C\sqrt{\xi}\ . \tag{116}$$

This represents a progressively increasing illumination from edge to edge as graphically illustrated in Fig. 16.

With the origin of co-ordinates at one edge of the slit as before, the integral (113) reduces to the form

$$I_{,,,} = A''' \int_0^{\sigma} \sqrt{\xi} \frac{\sin^2 \frac{\pi}{a_0}\left[a - \left(\xi + \frac{\sigma}{2}\right)\right]}{\left[\frac{\pi}{a_0}\left\{a - \left(\xi + \frac{\sigma}{2}\right)\right\}\right]^2} d\xi\ . \tag{117}$$

This integral has been evaluated for a slit width $\sigma = a_0$ as before for the special case $C = 1$. The results are tabulated and plotted in terms of $\frac{a}{a_0}$ as in the preceding case in Table VI and Fig. 17.

The position of maximum intensity, o′ is, in this case, displaced about $0.13\,a_0$ from the center of the geometrical image, *i. e.*, about twice the limiting error of measurement.

Cases in which the illumination of the slit is of the nature of that just investigated are frequently met with in spectrometric work on the Sun and planets when the image of these bodies is

placed eccentrically on the slit or when particular portions of the surface, such as the Sun-spots or solar prominences, are under examination. In the latter case the illumination is likely to be of the nature of that assumed in (114) and (115), and the effect of the variation in intensity of the image on the measured

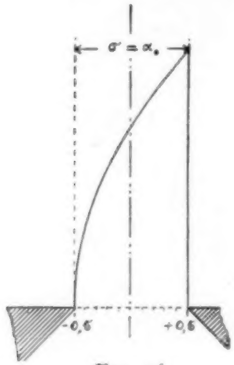


FIG. 16.

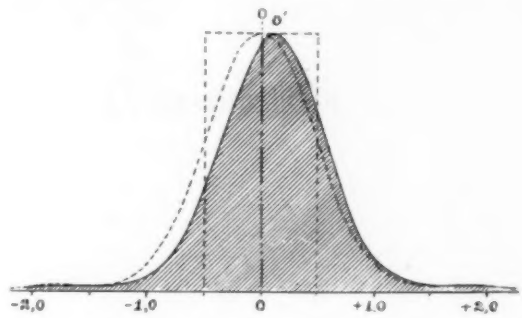


FIG. 17.

wave-lengths of the spectral lines can be disregarded, as we have seen, only when said variation is less than 50 per cent.

In the case of measurements of the velocity of rotation the slit is generally placed tangentially to the limb of the Sun or planet, and the conditions of illumination are then more gener-

TABLE VI.

$\frac{a}{a_0}$	$I_{...}$	$I' = f(\gamma)$[1]	$\frac{a}{a_0}$	$I_{...}$	$I' = f(\gamma)$[1]
−1.0	0.062	0.11	+0.00	0.969	1.00
−0.9	.090		+0.1	1.000	
−0.8	.136	0.24	+0.2	0.985	0.92
−0.7	.204		+0.3	.925	
−0.6	.295	0.45	+0.4	.827	0.71
−0.5	.406		+0.5	.702	
−0.4	.533	0.71	+0.6	.564	0.45
−0.3	.665		+0.7	.426	
−0.2	.784	0.92	+0.8	.301	0.24
−0.1	.896		+0.9	.198	
−0.0	.969	1.00	+1.0	.120	0.11

[1] For the purpose of comparison the values of I′, which expresses the distribution in intensity in the image of a slit of the same width uniformly illuminated, are given in columns 3 and 6. See *Phil. Mag.* **43**, 323, Table I; ASTROPHYSICAL JOURNAL, 3, 333 *et seq.* The curve $I' = f(\gamma)$ is plotted in dotted lines in both Fig. 15 and Fig. 17.

ally such as are represented by the distribution assumed in (116) and (117). For in the case of the Sun the variation in the intensity of radiation from the center to the edge is of the nature shown by the curves of Fig. 1 of the preceding paper of this series,[1] and may be represented by the general expression

$$i = \phi\,(\xi^2 + \eta^2)^{\frac{1}{2}}\ . \tag{118}$$

The effective intensity $f(\xi)$ at any point on the axis $\eta = 0$ will be found for this case by integrating the expression

$$f(\xi) = \int_0^{\sqrt{R^2-\xi^2}} \phi\,(\xi^2 + \eta^2)^{\frac{1}{2}} \frac{\sin^2 \frac{\pi}{a_0}(a-\eta)}{\left[\frac{\pi}{a_0}(a-\eta)\right]^2} d\eta\ . \tag{119}$$

At the edge of the Sun the value of η increases very rapidly with respect to ξ, and for large values of η the integral

$$\int_0^{\eta} \frac{\sin^2 x}{x^2}\, dx$$

rapidly approaches a constant value. Hence for the purposes of the present general investigation in which we are concerned more with the form of $f(\xi)$ than with its absolute value, we may neglect the second factor of the term under the integral sign of (119) and simply write

$$f(\xi) \cong \int_0^{\sqrt{R^2-\xi^2}} \phi\,(\xi^2 + \eta^2)\, d\eta\ . \tag{120}$$

This integral was evaluated by the writer several years ago (in connection with another research by Professor Michelson), for the particular value of $\phi\,(\xi^2 - \eta^2)$ corresponding to wave-length $\lambda = 5790$ tenth-meters, as given in Vogel's tables. The results were published in the form of a curve (Fig. 5) in Michelson's paper in the *Philosophical Magazine*.[2] An examination of this

[1] F. W. VERY, "The Absorptive Power of the Solar Atmosphere." *Misc. Sci. Papers*, Allegheny Observatory, No. 9; ASTROPHYSICAL JOURNAL, **16**, 73. See also VOGEL, "Spectralphotometrische Untersuchungen," *Monat. d. K. Akad. d. Wiss.*, Berlin, March 1877; pp. 23, 24, Plate 11.

[2] "Application of Interference Methods to Astronomical Measurements," **30**, 1.

curve will show that for small values of ξ the function $f(\xi)$ can be very closely represented by the parabola

$$f(\xi) = C\sqrt{\xi}$$

which is the form of function used in (116) and (117) already examined.

The effect of this inequality of illumination is, as we have already seen, to displace the spectral image of any line by an amount considerably in excess of the limiting accuracy of measurement. The exact amount of the displacement will depend (*a*) on the width of the slit in comparison with the diameter of the solar image, (*b*) on the position in which the latter is placed with reference to the slit opening. In the case already investigated in (117) we have assumed that the photospheric edge of the solar image is placed tangent to the outer edge of the slit so that when $\xi = o$, $f(\xi)$ is also zero.[1] If the image overlaps the slit, the conditions approximate more nearly those assumed in (114) and (115) and the displacement is less; if the edge of the image is within the slit, the displacement is even greater. Thus when the edge is coincident with the center of the slit, the position in which it is generally placed for velocity of rotation measurements, the displacement of the central point of intensity from the geometrical center will be nearly 0.6 of the half width of the slit, *i. e.*, about 0.3 a, or four times the limiting accuracy ϵ.

The direction of the displacement in the spectrum will depend on the position of the solar image with reference to the direction of deviation of light by the spectroscope train. If the edge of the image is on that side of the slit corresponding to the red end of the spectrum, the displacement will be toward the violet, and the wave-length derived from the measurement will be too small; if turned in the opposite direction the wave-length will be correspondingly too large. The sign of the displacement will, therefore, change not only as we shift the solar image so

[1] This of course is only an approximation to the actual conditions at the edge of the Sun. There is, however, such a sharp demarcation in intensity of total radiation between the photosphere and the chromosphere and outlying corona that the approximation is in most cases not far from the truth.

that first one edge and then the other falls on the slit; but also as we reverse the direction of deviation of the spectroscope train, "right" or "left," leaving the position of the solar image unchanged. If we therefore attempt to determine the velocity of rotation of a body by the direct measurement of the separation of the spectra from the opposite edges of the slit image, the total error in any one position of the spectroscope will be twice the error of displacement noted above. If we then take a corresponding series of measurements with the direction of dispersion reversed, the errors of displacement will be of the same magnitude as before, but reversed in sign; hence the total difference in measurement will be four times the individual displacement in any one position of image and spectroscope.

Errors of such magnitude as these cannot, of course, be disregarded, even though they may be eliminated by the exact reversal of the relative positions of image and spectroscope. I have, however, failed to find any reference to this particular source of error in the papers of the investigators who have given us our best spectroscopic determinations of velocity of solar rotation. Crew, indeed, in his second investigation[1] notes that the results of his measurements differ with grating "right" and grating "left," but he ascribes this difference to an entirely different cause, *i. e.*, the local heating of the slit jaws. This will undoubtedly produce an effect unless the greater part of the solar image is cut off by a suitable screen interposed in front of the slit, but even when this is not done the total displacement of the slit jaws due to this cause can hardly, in the opinion of the writer, be as great as it is necessary to assume to explain the observed differences of measurement. To do this it is necessary, as Crew has shown, to assume a change of temperature of about 15° C.[2] of each jaw between each measurement, in an interval of only about one minute, and to assume, further, that the expansion due to this change in temperature is all toward the

[1] "On the Period of Rotation of the Sun," *Am. Jour. Sci.*, 38, 204.

[2] The assumption of a difference of 10° C., made in Crew's paper, p. 210, explains only about two-thirds of the observed differences of equatorial velocities with grating "right" and grating "left."

slit opening. Crew observed actual differences of temperature of 10° between two thermometers placed on the two sides, but the change in the slit jaws themselves would be probably much less on account of their own mass and the masses of surrounding and attached metal. Likewise, the jaws when heated, would, if of the usual double-motion construction, expand in both directions. Lastly, if the change of temperature explanation were the correct one, there would be a continual and progressive shift of the spectral image during the entire interval of measurement, and this effect would not have been likely to escape the attention of so careful and accurate an experimentalist as Professor Crew.

On the other hand, if the width of the slit and the setting of the edge of the solar image corresponded to the conditions assumed in the previous integration of (117) the resulting displacement of the spectral images "right" and "left" would *fully* explain and account for the observed differences of measurement. For, as we have shown, the total difference of measurement to be expected is four times the individual displacement of any one image, and this latter is $0.13a_0$ for the conditions assumed. In this instrument the aperture and the focal length of the view telescope are 4 inches and 94 inches respectively. Hence for the D lines the linear value of a_0 is

$$a_0 f = \frac{0.0005896}{94} \times 4 = 0.014 \text{ mm} .$$

Hence the total difference $= 4 \times 0.13a_0 = 0.52a_0$ is in linear measure

$$0.014 \text{ mm} \times 0.52 = 0.0073 \text{ mm} .$$

The total observed difference in measurement amounted to about 0.015 revolutions of the micrometer screw, or, since the latter is of 0.5 mm pitch, to

$$0.0075 \text{ mm} ,$$

or almost exactly the quantity required by theory. When, therefore, the cause of displacement investigated above is taken into account in Crew's measurements, there is no discrepancy remaining to be explained.

The results already obtained indicate the care which it is necessary to take in determining the exact distribution of light in the slit image when the latter is necessarily variable, as in the preceding case. If this cannot be determined, the only way to avoid sensible errors in the wave-length measurements is to secure and maintain an exact centering of the image on the slit, so that the distribution of light over the slit opening is symmetrical about a center line. A case of great importance in this connection is that of determinations of wave-length in stellar spectra for the purpose of measuring motions in the line of sight, etc., with the compound slit spectroscope. The image of the star, as formed on the slit by the main telescope objective, is, if the latter is free from unsymmetrical aberration, a symmetrical diffraction pattern. If the slit is placed in the focal plane of the main objective, the theoretical distribution of light in this image is, for any given wave-length, represented by the usual expression (4), *i. e.*,

$$I_c^2 = A \frac{J_1^2\left(\frac{\pi}{a_o}\xi\right)}{\left(\frac{\pi}{a_o}\xi\right)^2} .$$

In long photographic exposures, the actual effective distribution of light in the slit image is somewhat different from this, owing to atmospheric and instrumental disturbances which broaden the image into what Newall has very aptly termed a "tremor disk." The determination of the exact distribution of light in such a disk is a matter of considerable uncertainty, since the broadening results not only from vibrations of the image, but also from temporary changes of focus and of chromatic dispersion and aberration due to passing air-waves of variable intensity. Under good conditions of "seeing," however, the principal cause of the broadening may be regarded as due to vibrations, and under such conditions we may derive an expression which will represent at least a closely approximate distribution of light in the tremor disk, from the law of probability.

The general expression which represents the probable law of

distribution of errors, or, in this case, of displacements from a central position, is

$$y = e^{-h^2\xi^2} \tag{121}$$

and the most probable error (*i. e.*, displacement) is that for which $y = 0.5$ and is given by the relation

$$\xi_0 = P = \frac{0.4769}{h} \tag{122}$$

In the present case it seems a fair assumption to consider that the most probable displacement of the image will be about $\frac{1}{2}\,a_0$, *i. e.*, half the resolving power of the main telescope objective. This assumption gives us for h

$$h = \frac{0.9538}{a_0}\ , \tag{123}$$

which substituted in (121) gives us for y

$$y = e^{-0.9097\frac{\xi^2}{a_0^2}}\ . \tag{124}$$

The resulting distribution in intensity in the tremor disk will then be represented by the integral

$$\int_{-\infty}^{+\infty} e^{-0.9097\frac{\xi^2}{a_0^2}} \frac{J_1\left(\frac{\pi}{a_0}\xi\right)}{\left(\frac{\pi}{a_0}\xi\right)^2} d\xi = f(\xi)\ , \tag{125}$$

or if we assume a rectangular aperture as before, by

$$I_t^2 = \int_{-\infty}^{+\infty} e^{-0.9097\frac{\xi^2}{a_0^2}} \frac{\sin^2\frac{\pi}{a_0}\xi}{\left(\frac{\pi}{a_0}\xi\right)^2} d\xi = f(\xi)\ , \tag{126}$$

and the resulting distribution in the spectral image of the slit of width σ_0 will be

$$I_{,,,}^2 = \int_{-(\sigma_2-\sigma_1)}^{+\sigma_1} I_t^2 \frac{\sin^2\frac{\pi}{a_0}(a-\xi)}{\left[\frac{\pi}{a_0}(a-\xi)\right]^2} d\xi\ . \tag{127}$$

In the integration of (126) and (127) it is convenient to express a_0 in terms of σ_0, the width of the spectroscope slit.

The usual practice is to make this width equal to $2\,a_0$[1] so as to just include the central diffraction disk of the undisturbed star image. Substituting this value of σ_0 in the above equations we obtain

$$I_t^2 = \int_{+\infty}^{-\infty} e^{-3.64\frac{\xi^2}{\sigma_0^2}} \frac{\sin^2 \frac{2\pi}{\sigma}\xi}{\left(\frac{2\pi}{\sigma}\xi\right)^2}\, d\xi \,. \tag{128}$$

This has been integrated by mechanical quadrature as before. The results are tabulated in Table VIII and plotted in Fig. 18.

TABLE VIII.

$\frac{a}{\sigma}$	I_t^2	$e^{-A'\xi^2}$	$I_t^2 - e^{-A'\xi^2}$
±0.00	1.000	1.000	0.000
±0.10	0.972	0.973	−.001
±0.20	.888	.892	−.004
±0.30	.771	.773	−.002
±0.40	.632	.633	−.001
±0.50	.490	.490	±.000
±0.60	.361	.358	+.003
±0.70	.254	.247	+.007
±0.80	.171	.161	+.010
±0.90	.111	.099	+.012
±1.00	.069	.057	+.012

The full curve, I_t, closely resembles the exponential curve $e^{-A'\xi^2}$. We have therefore substituted for (128) a second empirical curve of this form having such a value of A' that the two curves coincide at the point $x = a_0 = \frac{1}{2}\sigma_0$. This gives us for A'

$$A' = 2.854 \,,$$

and for $f'(\xi)$

$$e^{-2.854\frac{\xi^2}{\sigma_0^2}} \cong I_t^2 \,. \tag{129}$$

The values of $f'(\xi)$ from (129) are tabulated in the third column of Table VIII and plotted as the dotted curve in Fig. 18. Over all that part of the tremor disk which is likely to fall within the slit opening, *i. e.*, from $\xi = 0$ to $\xi = \pm 2\,a_0$, the coin-

[1] This is twice as large as the value of σ assumed in the previous cases. Compare Figs. 14, 16, and 18.

cidence between the two curves, I_t^2 and $f'(\xi)$, as given by (129), is closer than the uncertainty as to the exact form of I_t^2 itself.

Assuming that the effective size of the tremor disk is defined by the limits $m\,m$ such that $I_m^2 = 0.04 I_0^2$, we have under the conditions assumed above

$$m\,m = W \cong 4.5\alpha_0 = 2.25 m_0\,m_0 \; ; \qquad (130)$$

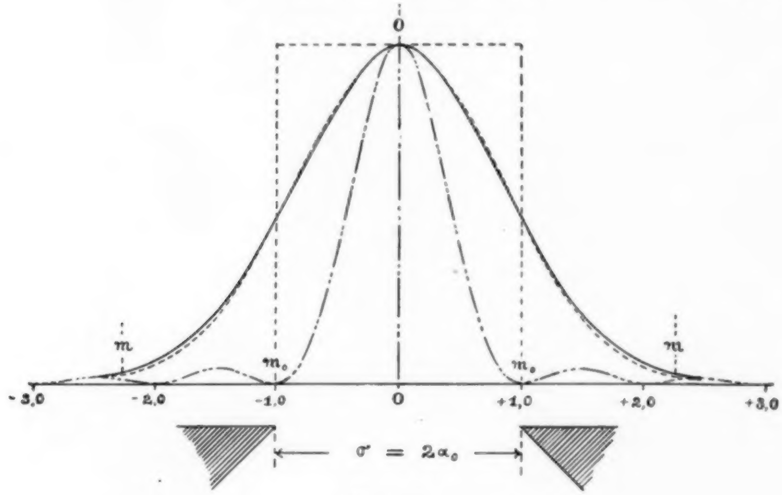


FIG. 18.

i. e., the tremor disk is about two and one-quarter times the size of the true central diffraction disk of the star. When the tremor disk is centered on the slit opening, the percentage of the total quantity of light in the image which enters the slit is of course given by the expression

$$\frac{2}{\sqrt{\pi}}\int_{-\frac{1}{2}\sigma}^{+\frac{1}{2}\sigma} e^{-2.854\frac{\xi^2}{\sigma_0^2}}\,d\xi \cong 0.76 \; ; \qquad (131)$$

i. e., the time of exposure is increased about one-third by the effect of atmospheric and instrumental vibrations of the previously assumed probable magnitude of $\frac{1}{2}a_0$.

This result agrees fairly well with the actual exposure times which Campbell finds necessary in obtaining well defined spectra

with the Mills slit spectroscope as compared with those required on the same stars with the Harvard objective prism spectrograph, when we allow for the difference in the resolving power and light efficiency of the two instruments. It is lower than the estimate given by Frost in his description of the Bruce spectrograph, but for some reason the latter is somewhat less efficient in regard to exposure times than the Mills instrument.[1]

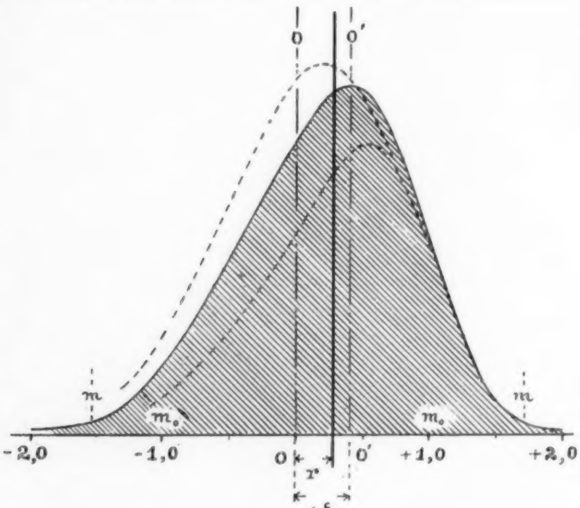


FIG. 19.

Substituting the value of $f'(\xi)$ given by (129) in (127), and expressing a_0 in terms of σ_0 as before, we obtain

$$I^2_{,,,} = \int e^{-2.854\frac{\xi^2}{\sigma_0^2}} \frac{\sin^2 \frac{2\pi}{\sigma_0}(a-\xi)}{\left[\frac{2\pi}{\sigma}(a-\xi)\right]^2} d\xi \ . \tag{132}$$

If, as before, we take the origin of co-ordinates at the point corresponding to the geometrical image of one edge of the slit and consider the most general case in which the center of the tremor disk may be located anywhere within the slit opening, we have as before for ξ

$$\xi = \xi' + \frac{\sigma}{2} + \Delta\sigma \ ,$$

[1] Compare result on p. 20 of Frost's paper, ASTROPHYSICAL JOURNAL, **15**, with those given by Campbell, ASTROPHYSICAL JOURNAL, **8**, 141.

where $\Delta\sigma$ is the amount by which the center of the tremor disk is displaced from the center of the slit. Substituting, this gives

$$I^2_{,,,}=\int_0^{\sigma_0} e^{-2.854\frac{(\xi+\frac{\sigma}{2}+\Delta\sigma)^2}{\sigma_0^2}} \frac{\sin^2\frac{2\pi}{a_0}\left[a-\left(\xi+\frac{\sigma}{2}+\Delta\sigma\right)\right]}{\left[\frac{2\pi}{a_0}\left\{a-\left(\xi+\frac{\sigma}{2}+\Delta\sigma\right)\right\}\right]^2}d\xi\ . \quad (133)$$

The only case for which $I^2_{,,,}$, (133), is symmetrical is that for which $\Delta\sigma = 0$. The center of intensity is then at the point $\xi = \frac{1}{2}\sigma_0$, and there is no error of displacement.

In order to find the relation between the error of centering, $\Delta\sigma$, and the resulting displacement $\Delta\xi$ of the spectral image, I have integrated (133) for different values of $\Delta\sigma$ from $\Delta\sigma = 0.1\sigma_0$ to $\Delta\sigma = 0.5\sigma_0$. The resulting values for $I^2_{,,,}$ are given in Table IX and the curves for $\Delta\sigma = 0.3\ \sigma_0$ (full) and $\Delta\sigma = 0.1\ \sigma_0$ and $0.5\ \sigma_0$ (dotted) are plotted in Fig. 19.

TABLE IX.

$\frac{a}{\sigma}$	$I_{,,,}$ for Different Values of $\frac{\Delta\sigma}{\sigma}$				
	$\Delta\sigma = 0.1\sigma$	$\Delta\sigma = 0.2\sigma$	$\Delta\sigma = 0.3\sigma$	$\Delta\sigma = 0.4\sigma$	$\Delta\sigma = 0.5\sigma$
−1.00			0.018		
−0.90			.021		
− .80			.034		
− .70			.058		
− .60			.107		
− .50			.191		
− .40			.296		
− .30			.422		
− .20			.550		
− .10			.678		
− .05	0.948		.742		
± .00	.977	0.909	.803	0.677	0.545
+ .05	**.995**	.948	.858	.741	.611
+ .10	**1.000**	.975	.903	.799	.672
+ .15	0.988	**.986**	.935	.844	.727
+ .20	.959	**.978**	[**.946**]	**.872**	.765
+ .25	.912	.948	.934	**.877**	**.783**
+ .30		.893	.896	.855	**.776**
+ .35		.817	.832	.807	.742
+ .40		.724	.746	.732	.682
+ .50			.532	.534	.519
+ .60			.315		
+ .70			.150		
+ .80			.061		
+ .90			.024		
+1.00			.016		

The positions ξ_0 of the points of maximum intensity in the curves $I^2_{,,,}$ are found to be as follows:[1]

For $\Delta\sigma = 0.1\sigma_0$, $\xi_0 \cong 0.595\sigma_0$ $\Delta\xi \cong 0.095\sigma_0 = 0.19a_0$
$\Delta\sigma = 0.2\sigma_0$, $\xi_0 \cong 0.655\sigma_0$ $\Delta\xi \cong 0.155\sigma_0 = 0.31a_0$
$\Delta\sigma = 0.3\sigma_0$, $\xi_0 \cong 0.700\sigma_0$ $\Delta\xi \cong 0.200\sigma_0 = 0.40a_0$
$\Delta\sigma = 0.4\sigma_0$, $\xi_0 \cong 0.735\sigma_0$ $\Delta\xi \cong 0.235\sigma_0 = 0.47a_0$
$\Delta\sigma = 0.5\sigma_0$, $\xi_0 \cong 0.765\sigma_0$ $\Delta\xi \cong 0.265\sigma_0 = 0.53a_0$.

The relation between $\Delta\sigma$ and $\Delta\xi$ is plotted in Fig. 20 (full curve). As will be seen by the dotted curve it can be represented very closely by the empirical equation

$$\Delta\xi = \frac{7}{8} a_0 \sqrt[3]{\left(\frac{\Delta\sigma}{\sigma}\right)^2} . \tag{134}$$

In measuring the apparent position of the displaced diffraction image the tendency will be, as before, to compromise between the point of maximum intensity, o', and the point midway between the apparent edges, $m\ m$, (defined as before), of the line. The resultant setting, r, for the cross-wire will be at a point not far from $\frac{2}{3}\Delta\xi$. We may therefore write

$$\left.\begin{aligned} \Delta_m\xi &= \text{measured displacement} \cong \frac{2}{3}\Delta\xi \\ &\cong 0.58 a_0 \sqrt[3]{\left(\frac{\Delta\sigma}{\sigma}\right)^2} \end{aligned}\right\} . \tag{135}$$

In order that the displacement shall not be great enough to affect the accuracy of measurement we must have as before

$$\Delta_m\xi \leqq \epsilon ,$$

or in this case from (14) and (135)

$$0.07 \geqq 0.58 \sqrt[3]{\left(\frac{\Delta\sigma}{\sigma}\right)^2} , \tag{136}$$

or

$$\frac{\Delta\sigma}{\sigma_0} \leqq .04 .$$

i. e., the star image must be centered on the slit with an error not exceeding 4 per cent. of the slit width.

This is a condition which has apparently never before been

[1] The points of maximum lie between the numbers in heavy type in the table.

recognized as necessary to accuracy in the use of the slit spectroscope. Care has of course always been taken to keep the star image as nearly central as possible in the slit opening, but the object aimed at has been simply to utilize the greatest amount of light and shorten the time of exposure as much as possible. But the above investigation shows that the requirements of accuracy and the avoidance of constant errors require a far greater degree of care in centering and following than is usually deemed necessary, greater in fact than many of the devices now used for following make possible.

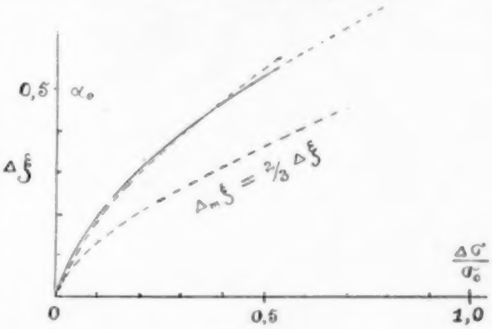


FIG. 20.

This question is one of such great importance in view of the minuteness of the displacements upon which determinations of motions in the line of sight depend, that it may be interesting to calculate the error in kilometers per second produced by an error of say 10 per cent. in the centering of the star image on the slit. From the previous results we find, for $\frac{\Delta\sigma}{\sigma_o} = 0.10$,

$$\Delta_m \xi = 0.13 a_o \ . \tag{137}$$

Since the angular resolving power a_o is the same for both collimator and view telescope, we can readily establish a relation between a_o, $\frac{d\theta}{d\lambda}$ = the dispersion of the spectroscope train, and v_s the motion in the line of sight of the body under examination. From the well-known relations

$$v_s = \frac{\Delta\lambda}{\lambda} V$$

$$f a_o = \frac{\lambda}{b} f = \frac{\lambda}{\beta}$$

and

$$\Delta\lambda = \frac{\Delta\xi}{\frac{d\theta}{d\lambda}} ,$$

we get at once
$$v_s = \frac{\Delta\xi}{\frac{d\theta}{d\lambda}} \cdot \frac{V}{\lambda},$$

and for the value of $\Delta_m\xi$ assumed in (137)
$$\delta v_s = \frac{0.13V}{b\frac{d\theta}{d\lambda}} = \frac{0.13V}{R}$$
$$= \frac{38000}{r} \text{ km per sec.}, \tag{138}$$

where r is the spectroscopic resolving power of the prism train.

The following table contains the approximate resolving powers of the more important instruments that have been used in line of sight work and the corresponding error δv_s produced by a 10 per cent. error of centering of the star disk from (138):

TABLE X.

Name of Instrument	Spectroscope Train	Approximate r for $H\gamma$	δv_s Kilometers per Second
Potsdam II (Vogel)	2 Compound Prisms	about 40,000	about 0.9
Pulkowa (Bélopolsky)			
Bruce I (Newall)[1]	1 Simple Prism	22,000	1.7
Mills (Campbell)[2]	3 " Prisms	74,000	0.5
McMillin (Lord)[3]	2 " "	42,000	0.9
Potsdam III (Vogel)[4]	3 " "	63,000	0.6
Bruce II (Frost)[5]	3 " "	110,000	0.2

An examination of this table shows that even for this small error in centering the errors in the resulting determinations of v_s are sufficiently large to account for a large part of the differences of measured velocities between some of the different observers. When we remember the difficulty of keeping a faint star *accurately* centered on the slit, and remember, also, that any constant asymmetry of the tremor disk itself due to peculiar instrumental conditions will notably increase the effect of a given displacement from the center, we can readily see that we have here what may be a constant source of error of the most serious kind, and in some cases of even greater magnitude than indicated in the above table. If, for example, the error of centering were

[1] ASTROPHYSICAL JOURNAL, **3**, 266.
[2] *Ibid.*, **8**, 123.
[3] *Ibid.*, **4**, 50.
[4] *Ibid.*, **11**, 393.
[5] *Ibid.*, **15**, 1.

for any reason constant in direction with reference to the large telescope, any reversal of the spectroscope with reference to the latter would introduce differences in observed positions of the lines, and hence of v_s of twice the amount indicated in Table X.

As this error is purely one of manipulation, we cannot in general make any correction or allowance for it *after* the photographic record has been taken. The utmost care, therefore, must be exercised in order to attain the accuracy of centering and following requisite to eliminate it completely, as indicated in (136). To do this we should, as far as possible, fulfill the following general conditions:

1. Have a sharp, well-defined, and symmetrical image of the star formed on the slit of the spectrograph. If the main image-forming objective is a visual refractor, this will necessitate in general the use of a correcting lens, and this should be so designed as to be free from spherical and chromatic aberration for the region of the spectrum under examination, and so mounted that its principal optical axis *coincides* with that of the main objective and the axis of collimation of the spectroscope itself. The light from the other portions of the spectrum should be cut out from the following eyepiece by use of suitable screens or screening devices [1] placed between the slit and eyepiece. On account of the perfect achromatism of the reflecting telescope this form of objective has great advantages over the refractor in this connection; another reason why such instruments should be given the preference for spectroscopic work.[2]

2. The guiding device should use as a reference mark for centering the star image, some point on the slit itself, in order

[1] Such, for example, as the optical color screen described by the writer in this JOURNAL, **3**, 169.

[2] The importance and value of the reflecting mirror as compared with the refractor in spectroscopic work have been urged upon the attention of astrophysicists by the writer for many years. See *Phil. Mag.*, July and October, 1894; *A. and A.*, December, 1894; ASTROPHYSICAL JOURNAL, January, 1895; *ibid.*, March, 1895; *ibid.*, March, 1896; *ibid.*, May, 1896; *ibid.*, October, 1896; *ibid.*, February, 1897; *ibid.*, February, 1898; *Pop. Astron.*, February, 1898, etc. The advantages of the reflector in this line of work are now being more generally recognized, and a number of large instruments of this type are planned or in actual course of construction by our large astrophysical observatories.

to avoid any errors of parallax or relative displacement of the slit and guiding cross-wires. For this reason I believe a modification of the Huggins reflecting slit and following device is better adapted to this purpose than any other form yet invented. The use of an auxiliary independent guiding telescope such as is proposed with the new Potsdam instrument would seem to be especially dangerous.

3. If the required accuracy of following cannot be attained with the usual slit width $\sigma_0 = 2\,a_0$, then the width should be reduced and the time of exposure correspondingly increased, or else greater spectroscopic resolving power should be used. The effect of decreased slit width will be to reduce the limits of integration in (127) and (133), and correspondingly reduce the resultant asymmetry and displacement of the center of intensity of the spectral image. The result of increasing R will be to decrease the value of δv_s in (138) for a given value of $\Delta\sigma$.

In this connection it may also be noted that "bad seeing," which results in an increase in size of the tremor disk, will reduce the effect of a given percentage error of centering $\frac{\Delta\sigma}{\sigma_0}$ by flattening the curve I_t^2 of Fig. 18, and thus reducing the difference between the intensity of illumination at different parts of the slit image. When the "seeing" is "bad," therefore, the slit may be opened wider without increasing the effect of errors of following.

B (4). It was the intention of the writer to take up also, in connection with the present paper, the discussion of effects of errors of mechanical design and construction, with reference particularly to the avoidance of flexure and strain resulting from changes in position or changes of temperature of the instrument during use. The present discussion has, however, so far exceeded the limits originally set that it seems better to reserve this part of the subject for a future paper in which the description of the spectrographs and some other astrophysical instruments of the new Allegheny Observatory will also be taken up.

ALLEGHENY OBSERVATORY, 1901–1902.

ON SCREENS TRANSPARENT ONLY TO ULTRA-VIOLET LIGHT AND THEIR USE IN SPECTRUM PHOTOGRAPHY.

By R. W. Wood.

Anyone who has repeated Tyndall's beautiful lecture experiment of kindling a pine stick in the dark heat focus of a burning-glass, concentrating light from which the visible radiations have been removed by means of a solution of iodine in bisulphide of carbon, must have wished that we possessed a screen opaque to visible light and transparent to the ultra-violet.

I have recently succeeded in making a screen quite transparent to these radiations, though a gas flame cannot be seen through it. By combining it with a large condensing lens and an arc-lamp, it is possible to form a dark focus of ultra-violet light in which a lump of uranium nitrate glows with a vivid green phosphorescence like a great emerald.

Aside from giving us the means of performing a most beautiful lecture experiment, these screens make it possible to photograph the ultra-violet lines in grating spectra of higher orders than the first, entirely uncontaminated by the visible radiations which overlie them. Other applications at once suggest themselves, such as the complete removal of the highly actinic blue and violet rays, in certain investigations of the ultra-violet region where the long exposures necessary are apt to produce fogging of the plates. It seems quite possible, too, that photographs of the Moon, planets, and nebulæ, taken by means of ultra-violet light, may furnish valuable data, as I shall attempt to show at the end of this paper.

The substance which has made possible the production of such a screen is nitroso-dimethyl-aniline, the remarkable optical properties of which I have already alluded to in a previous paper. As I have already said, a prism formed of this substance yields a spectrum about thirty times as long as a quartz prism of the same angle, the dispersion somewhat resembling that of selenium. I

was of the opinion that the absorption, which commences at about wave-length 0.0005 mm, would increase continuously from this point down to the end of the spectrum, as was found to be the case with selenium. On commencing a study of the absorption, however, I was astonished to find that it ended abruptly a little beyond the H and K lines, and that from this point on the substance was transparent even down to the last cadmium line, of wave-length 0.0002 mm. It at once occurred to me that, if some substance or substances could be found, absorbing the red, yellow, and green, and transparent to the ultra-violet, we could, by combining them with the nitroso compound, produce the long-sought screen.

Very dense cobalt glass, coated with a thin film of gelatine lightly stained with the nitroso, was found to be transparent only to the extreme red and the ultra-violet, and the red was eventually removed by means of a thin sheet of Chance's "signal-green" glass, such as is used for one of the reflectors in the Ives Kromskop. This combination is wholly opaque to visible light, while freely transmitting everything between wave-lengths 34 and 38. Of course, the employment of glass screens limits the ultra-violet transmission, and a screen of this description is useful chiefly for lecture demonstrations. Considerable care must be used in the adjustment of the strength of the solution of the nitroso in gelatine, otherwise the intensity of the ultra-violet light is considerably weakened. The best strength is such as will be just sufficient to remove the blue and violet light transmitted by dense cobalt glass. Quite a number of trials will be found necessary in adjusting the densities of the three components of the screen to secure the maximum effect, but when the balance is just right, it is possible to form a focus in which a piece of paper is quite invisible, while a mass of crystals of the nitrate of uranium (which I have found superior to anything else) glows with sufficient intensity to be seen from the back of the largest lecture-room. It is best to exclude carefully all light which does not pass through the screen.

With the assistance of one of our students, I am at the present time investigating the absorption of a large number of sub-

stances which, so far as I know, have not been previously studied, and I hope in time to dispense with glass entirely, and produce an opaque screen which transmits ultra-violet light down to the end of the spectrum. A tube filled with iodine vapor and furnished with quartz ends, on one of which is a thin film stained with nitroso, transmits all the ultra-violet, and only the extreme red, but it is very inconvenient to work with. For use as a screen in spectrum photography there is no especial object in removing the red, yellow, and green, the nitroso alone blocking out completely the actinic portions of the visible spectrum, which overlie the ultra-violet in the second, and third-order spectra, and I shall next consider solutions of the substance in various fluids in connection with spectrum photography.

I have found that the best method of quickly securing a record of the absorption of a solution is to bring a prismatic layer of the liquid, contained in a quartz cell, before the slit of a quartz spectrograph, and photograph the spectrum of the cadmium spark. We secure in this way a record of the absorption of the liquid in various thicknesses, in the form of a curve, quite similar to the curves laboriously constructed from the readings obtained with the spectro-photometer.[1]

The curve obtained with a solution of the nitroso in glycerine is shown in Plate XII, Figs. 4 and 5. It will be noticed that after a certain thickness has been passed we begin to get a noticeable absorption in the ultra-violet, the form of the curve in this region being well shown in Figs. 3 and 4. The band in the blue and violet is, however, so much heavier that, by employing a film of suitable thickness, we can get complete opacity in this region, combined with almost perfect transparency in the ultra-violet.

[1] It is my intention to prepare a monograph on the absorption of a large number of the aniline dies, and other organic compounds such as the nitroso-dimethyl-aniline' which have not been previously investigated. The spectra will be approximately normal, all on the same scale, and will extend from the C line down to the end of the spectrum. They will be photographed in the manner which I have described, and will, I hope, make it possible for the spectroscopist or physicist to pick out at once the combination necessary to produce any desired result. Preliminary experiments are now in progress to determine the best form to give the apparatus, and the most suitable source of light, and I shall be very glad of any suggestions pertaining either to the apparatus or to particular substances worthy of investigation.

The nitroso is soluble in water, glycerine, ether, alcohol, bisulphide of carbon, and many other fluids, and the region of heaviest absorption varies somewhat with the nature of the solvent, the shift of the band not, however, following Kundt's rule in every case. A stained gelatine film on a quartz plate forms a fairly suitable screen, if we do not wish to photograph below the group of cadmium lines at wave-length 2314. It is opaque, however, to waves much shorter than this. The glycerine solution transmits down to the last cadmium line, $\lambda = 2147$, and some other solvents appear to work equally well.

In photographing the spectrum of the cadmium spark in the ultra-violet of the third order, with the fourteen-foot concave grating, I found that the prolonged exposure of the solution in glycerine to the light of the spark resulted in its decomposition. Gas bubbles formed in the thin quartz cell, and, by bridging across the space between the two plates, allowed the passage of blue and violet light. The same thing occurred with pure glycerine under a quartz plate, while glycerine under glass was unaffected, showing that the decomposition was caused by the extreme ultra-violet. In addition to the formation of bubbles a gradual bleaching of the solution occurred. To obviate this difficulty I constructed a small cell of quartz, by cementing two plates together, with a space of about 0.5 mm between them, the cell thus formed being cemented to the bottom of a small thistle tube with a very small bore. By filling the thistle tube with the glycerine solution, a flow took place through the cell at the rate of about a drop every two minutes. This device worked admirably and gave no trouble at all, the cell being placed close to the slit of the grating camera in the path of the convergent beam from the quartz lenz. Another very satisfactory screen can be made by dissolving celluloid (previously boiled for some time in water) in amyl acetate, adding a little nitroso, and flowing the solution on a quartz plate. It is, however, opaque to the last two cadmium lines. The use of the screen necessitates considerable increase in the time of the exposure, the amount varying from two to ten, or even twenty, times, according to the density of the screen. The strength of the glycerine solution must be

PLATE XII.

5089 3820 3250 2748 2572 2314 2175

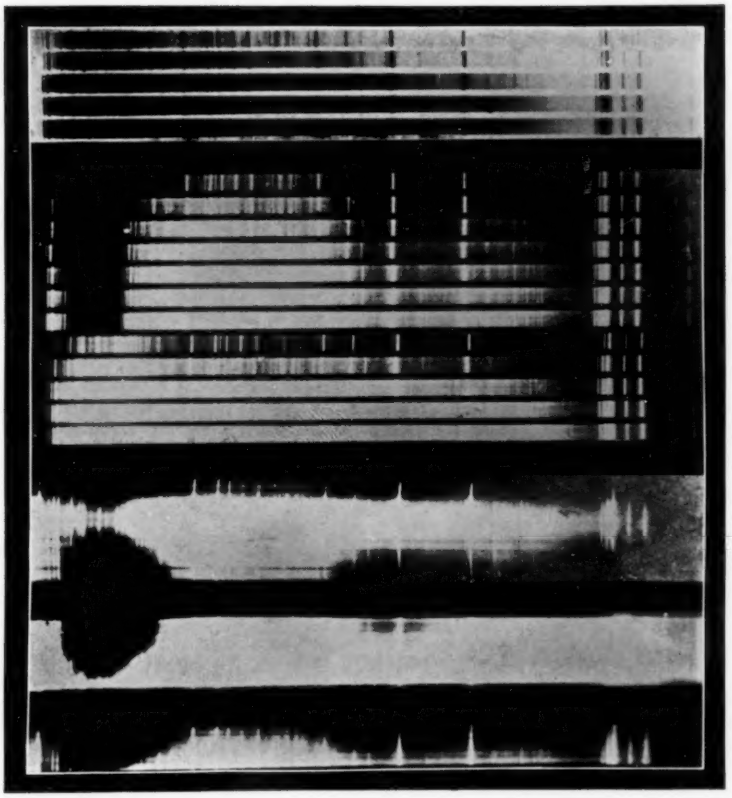
1

2

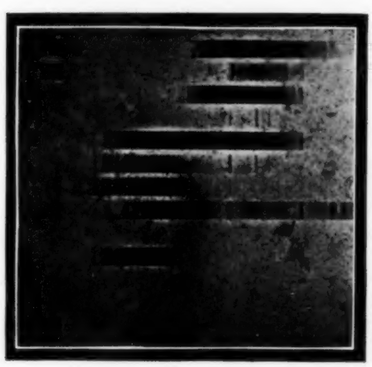
3

4

5



6



1
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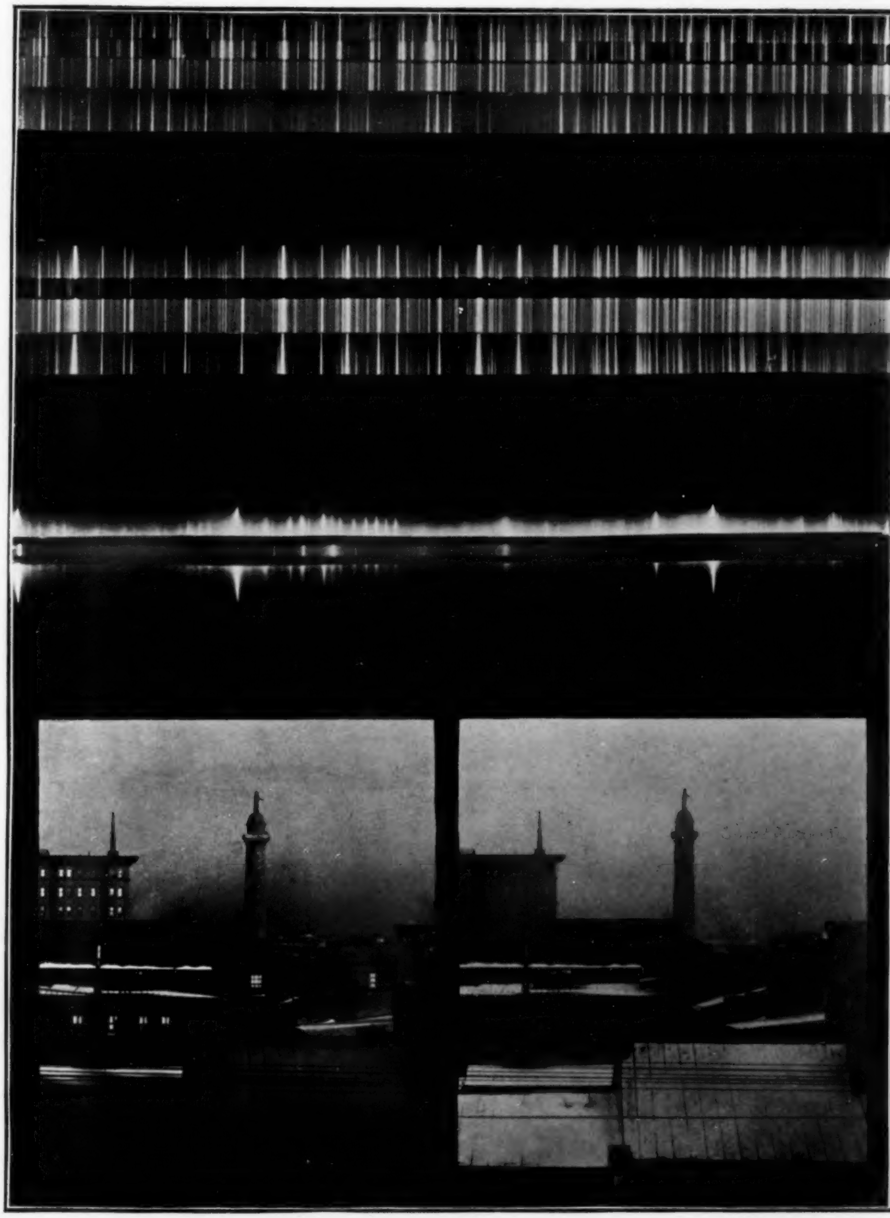
11

EFFECTS OF ABSORBING SCREENS.

PLATE XIII.



A B

Effect of Screens Transparent only to Ultra-Violet Light.

adjusted according to the work required of it; a strong solution gives a wider band in the blue and violet, but diminishes the intensity of the ultra-violet as well. In general, the best results are obtained when the blue line of wave-length 4799 in the spark spectrum of cadmium can be just barely discerned.

In Plate XII, Fig. 1, the wave-lengths of the principal lines in the spark spectrum of cadmium are given for reference. The action of the nitroso screen is well shown in Fig. 2, the spectra being photographed with a small quartz spectrograph made by Fuess. The first seven spectra were taken through the glycerine nitroso cell which I have just described, with the following times of exposure: 5, 10, 15, 20, 30, 45, and 60 seconds. The cell was then removed and the following six spectra taken with exposures 2.5, 5, 10, 15, 20, and 30 seconds. A study of these spectra enables us to calculate just what can be done with this screen, and the necessary increase in the time of exposure resulting from its use.

In Fig. 6, which is a negative, we have the absorption spectra of the various components of the screen which I mentioned in the first part of this paper, taken with exposures of 20 seconds each. The spectra were taken through screens as follows:

1. Nitroso in gelatine on glass (thin film).
2. Nitroso in gelatine on glass (thick film).
3. Nitroso in gelatine on cobalt glass (thin film).
4. Nitroso in gelatine on cobalt glass (thick film), strong ultra-violet absorption.
5. Dense cobalt glass.
6. Turnbull's blue in gelatine.
7. Chance's "signal green" glass (two thicknesses).
8. No screen, 3 seconds' exposure.
9. Cyanine in Canada balsam.
11. Aurantia in collodion.

These photographs were taken on an orthochromatic plate, the yellow and yellow-green being compressed into the small strip which appears alone in No. 11.

The utility of the nitroso screen in photography with the concave grating is very clearly brought out in the photographs of the iron spectrum shown in Plate XIII. These were made with

a fourteen-foot grating, with a glycerine nitroso cell before the slit during one of the exposures. Figs. 1 and 2 are from the same plate. Strip B in each was made through the screen, and shows the ultra-violet of the third order, uncontaminated by the blue of the second. In strips A, which were made without the screen, the two orders are mixed. Strips C were made through a glass screen, which cut off the third order ultra-violet, leaving the blue of the second. I have marked a few of the wave-lengths to aid in the identification of the lines. The times of exposure were for strips A and C 10 minutes, for B 50 minutes.

The group of cadmium lines in the neighborhood of wave-length 2314 is, in the second-order spectrum, mixed up with a lot of blue air lines of the first-order spectrum. The separation of the two by the nitroso screen is well shown in Fig. 3, in which the two orders are shown superposed in strips A, and the ultra-violet of the second order in strip B. The exposures in this case were 15 minutes and 2 hours respectively.

Another screen which I believe may prove useful in astro-physical work is made by combining nitroso-dimethyl-aniline with a small amount of the dye uranine, the latter removing the bluish-green portion of the spectrum which affects the photographic plate. By a proper adjustment of the two in gelatine on glass, a screen can be formed which, when used with an ordinary (*i. e.*, not orthochromatic) plate, gives us a photograph made exclusively by ultra-violet light, comprised between wave-length 345 and 365—a rather narrow range. I have made a few photographs with a screen of this description which have brought out some interesting points. In a photograph of the full Moon, taken by ultra-violet light, the contrast between the bright and dark areas is very strongly accentuated, while in photographs of landscapes made in the same way there is almost no contrast at all, except between white objects and objects not white. I have also photographed a collection of rocks and minerals with ultra-violet light and with yellow light. In the negative taken by yellow light there is a great deal of contrast and detail, especially in the marbles and conglomerates, while

in the negative taken by ultra-violet light all this is absent, the white specimens coming out very black, with everything else of a thin and almost uniform gray. I hope in the near future to have an opportunity of making some lunar photographs on a large scale, the only instrument at my disposal at the present time being the nine-inch equatorial of the University. Photographing by ultra-violet light appears to diminish the contrast between all objects not white, and to increase the contrast between white objects and those not white. I do not wish to be hasty in drawing conclusions, but it appears to me to be probable that the more luminous portions of the lunar surface, if not as white as plaster of Paris, must at least be much whiter than gray sandstone.

In Plate XIII, Fig. 4, are reproduced two photographs of the same landscape, taken at the same time and under similar conditions of illumination, the one (A) taken on an orthochromatic plate by yellow light through a screen of dense aurantia, the other (B) taken on an ordinary plate by ultra-violet light. The absence of contrast between the chimneys and walls in B is especially noticeable in the right-hand part of the picture. I tried various times of exposure, and the picture reproduced is the best of the lot. Another curious effect is the almost complete absence of shadows in the ultra-violet picture (it was taken in full sunlight like the other), showing that most of the ultra-violet light comes from the sky, which is what we should expect, though we should hardly anticipate that the effect would be so pronounced. This is best seen on the monument and on the snow in the middle distance. The increase of "atmosphere" in the ultra-violet picture is very marked. It is so strong that under-exposed plates fog in the shadows of objects not over one hundred yards from the camera, a circumstance which shows the great scattering power of the air for these short waves. The two pictures are also interesting as showing that our eyes have developed a maximum sensibility for that region of the spectrum which shows terrestrial objects in strongest contrast. Nitroso-dimethyl-aniline is the only substance, other than the ordinary aniline dyes, that I have examined thus far, and I feel

very hopeful of finding among the large number of allied substances, absorbing media even more transparent to the ultra-violet radiations than the one which I have described in this paper.

PHYSICAL LABORATORY,
JOHNS HOPKINS UNIVERSITY,
Baltimore, January 1903.

SYSTEMATIC ERRORS IN THE WAVE-LENGTHS OF THE LINES OF ROWLAND'S SOLAR SPECTRUM.

By G. EBERHARD.

In their investigations with their interference apparatus Messrs. Fabry and Perot[1] have measured the wave-lengths of 33 lines of the solar spectrum, and have compared their values with those given by Rowland in his preliminary table of solar spectrum wave-lengths. In this way they have shown that Rowland's system contains systematic errors of considerable amount in comparison to the accuracy of the relative measurements for small regions of spectrum. They give for a number of lines the ratio $\frac{\lambda \text{ (Rowland)}}{\lambda \text{ (Fabry and Perot)}}$, which in the absence of systematic errors should be constant, and for the sake of the subsequent comparison I have also plotted the numbers of their table on a curve given below.

Although any doubt as to the reality of these systematic errors in Rowland's table is wholly dispelled by the great accuracy attained by Messrs. Fabry and Perot in their investigations, it nevertheless seemed to me to be of interest to examine another independent series of observations—the wave-lengths of 300 solar lines measured by Müller and Kempf—and see whether a comparison of their values with Rowland's would exhibit the same thing. In a similar comparison Müller had already found that "in the central portions of the spectrum between wave-lengths about 550 and 610, negative signs prevail, which leads to the inference of systematic inequalities in one of the two series of measures," etc.[2] I had to take into account the fact that, in consequence of the nature of the grating then employed, which was decidedly inferior in quality to modern gratings, accidental errors of measurement would occur of much larger amount than in the case of Rowland, and that these could only be in

[1] ASTROPHYSICAL JOURNAL, **15**, 270–273, 1902.

[2] *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, **8**, 51.

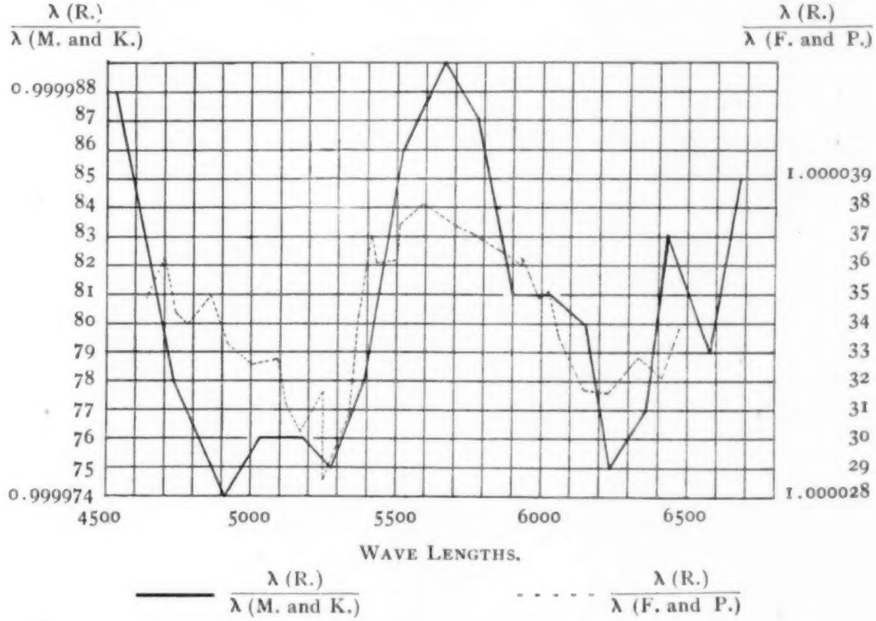
some measure compensated for by taking the means from a large number of lines.[1] But, on the other hand, Müller and Kempf proceeded with the greatest possible care in building up their system, so that the occurrence of systematic errors, which were dependent upon the wave-length, did not seem to me probable, aside from those errors which might arise from the determination of the constant of the grating.

On carrying out the computations, my suspicions were fully confirmed, as I found that the quotient $\frac{\lambda\ (\text{Rowland})}{\lambda\ (\text{Müller and Kempf})}$ exhibited very nearly the same course as the similar quotient between Rowland and Fabry and Perot; but the values of the first quotient disclose very much greater differences for neighboring lines than in the case of Fabry and Perot.

λ (Rowland)	λ (Rowland) / λ (M. and K.)	λ (Rowland)	λ (Rowland) / λ (M. and K.)	λ (Rowland)	λ (Rowland) / λ (M. and K.)	λ (Rowland)	λ (Rowland) / λ (M. and K.)
4494.738	1.000001	5281.971	0.999967	5775.304	0.999990	6246.535	0.999970
4501.448	0.999986	5283.802	0.999976	5806.950	0.999983	6252.773	0.999964
4508.455	1.000000	5288.705	0.999973	5816.601	0.999988	6256.572	0.999986
4563.939	0.999971	5307.541	0.999978	5831.821	0.999976	6265.348	0.999979
4572.156	0.999984	5324.373	0.999980	5862.582	0.999987	6318.239	0.999971
4703.177	0.999993	5353.571	0.999974	5896.155	0.999984	6322.907	0.999976
4754.225	0.999963	5367.669	0.999977	5934.881	0.999982	6335.554	0.999974
4861.527	0.999977	5383.578	0.999981	5948.765	0.999972	6344.371	0.999980
4890.948	0.999969	5389.683	0.999973	5977.007	0.999983	6355.246	0.999966
4903.502	0.999974	5393.375	0.999964	5987.290	0.999982	6358.898	0.999986
4920.685	0.999979	5405.989	0.999987	6003.239	0.999985	6380.958	0.999973
4924.107	0.999971	5415.416	0.999981	6013.715	0.999981	6393.820	0.999984
4973.281	0.999976	5434.740	0.999987	6024.281	0.999983	6408.233	0.999982
4980.352	0.999966	5487.959	0.999980	6042.315	0.999976	6411.865	0.999982
4999.689	0.999978	5497.735	0.999983	6056.227	0.999977	6421.570	0.999977
5050.008	0.999992	5501.683	0.999975	6065.709	0.999983	6431.066	0.999992
5090.954	0.999968	5528.641	0.999993	6078.710	0.999980	6439.293	0.999987
5133.870	0.999975	5543.414	0.999995	6102.937	0.999980	6450.033	0.999977
5159.231	0.999967	5555.122	0.999991	6122.434	0.999994	6534.172	0.999980
5162.449	0.999971	5624.769	1.000001	6141.938	0.999983	6546.479	0.999972
5172.856	1.000001	5634.171	0.999995	6162.390	0.999977	6563.045	0.999986
5183.791	0.999973	5675.647	0.999980	6180.420	0.999977	6609.360	0.999979
5215.353	0.999960	5688.436	0.999987	6200.527	0.999970	6633.995	0.999979
5233.122	0.999983	5731.984	0.999985	6213.644	0.999978	6643.876	0.999988
5242.658	0.999983	5754.881	0.999996	6219.494	0.999981	6678.235	0.999982
5269.723	0.999967	5763.218	0.999997	6230.943	0.999968	6717.940	0.999984

[1] Müller and Kempf give as the probable error of a line on the average ±0.03 tenth-meters (*Publicationen des Astrophys. Obs.*, **8**, 145), but it may be seen from the table, pp. 147 ff., that the probable error of a line depends very much upon its character.

I employed as the basis of my computation the list of 127 lines of the Potsdam list of wave-lengths which Müller[1] had already selected in making his comparison of the Potsdam values with Rowland's standards of 1889. Of these 127 lines I omitted, however, 23 more, as the Potsdam values of the line might have been affected by close and often fainter lines, in conse-



quence of the small dispersion and resolution employed in the measurements. Although it would have been desirable to have as great a number of common lines as possible for my comparison of the two systems, I nevertheless refrained from identifying a larger number of the 300 lines of the Potsdam list[2] with those of Rowland, as I have never made extensive observations with the old Potsdam glass grating, and therefore could not have made an advantageous selection of the lines. Rowland's values are of course taken from his "Preliminary Table."

The individual values of the above table exhibit a somewhat irregular behavior, but on plotting on section paper a progres-

[1] *Ibid.*, 49, 50. [2] *Ibid.*, 5, 147 ff.

sion is also clearly visible. This becomes more evident on combining into a mean the values of the ratio $\frac{\lambda\,(\text{Rowland})}{\lambda\,(\text{Müller and Kempf})}$ for lines 20 tenth-meters apart. But also in taking this mean still far too few single values are combined in each mean, and I therefore also took means for every 100 tenth-meters, which are represented in the accompanying curve. The close agreement with the curve of Fabry and Perot cannot fail to be recognized, and hence the occurrence of systematic errors in Rowland's table is also evident from the Potsdam determinations of wave-lengths by Müller and Kempf, in the manner first established by Fabry and Perot.

ASTROPHYSIKALISCHES OBSERVATORIUM,
Potsdam, January 3, 1903.

PRELIMINARY NOTE ON SOME MODIFICATIONS OF THE MAGNESIUM LINE AT λ 4481 UNDER DIFFERENT LABORATORY CONDITIONS OF THE SPARK DISCHARGE.

By Sir William Huggins and Lady Huggins.

In his "Note on the Wave-Length of the Magnesium Line at λ 4481"[1] Professor Crew points out that an interesting problem still remains, namely to discover the laboratory conditions under which the line becomes sharp, as in some stellar spectra.

For some years, at intervals, experiments have been made here in the laboratory on the spectrum of magnesium, with the hope of throwing light on the physical conditions of the stellar atmospheres which we may assume to be indicated by the character of this line when present; a line which in the laboratory is subject to a very wide range of modifications, both of character and of intensity.

As it may be some time before these experiments are sufficiently complete for publication, it seems desirable to reproduce at once with this preliminary note, out of the very large number of spectra which have been taken, a few representing the most typical forms of the modifications of this line.

The teaching of these experiments suggests that the condition of the spark-discharge which is most potent in bringing about modifications of this line both in intensity and in character is the greater or less suddenness of the blow of the discharge. To a small extent only does the character of the line appear to be affected by the quantity and the electro-motive force of the electricity which is in action; indeed, such changes as may appear are probably brought about indirectly by the larger mass of material acted upon as the discharge is made more powerful.

The appearance of the line at λ 4481 in the spectrum at the top of the plate may be taken as representing its normal condi-

[1] Astrophysical Journal, **16**, 246, 1902.

tion with capacity in the secondary of the coil. When the jar is taken out of circuit, and the discharge of the secondary takes place directly between the magnesium electrodes, the line becomes thin, defined, and of small intensity, as in spectrum No. 2. In this case the electric blows are less sudden through the incoming of the full self-induction of the coil itself.

The researches of Schuster, Hemsalech, Schenck, Huff, and others have shown that a similar effect follows when the jar-discharge is slowed down by the introduction into the circuit of an independent self-induction. The condition of the line in spectrum No. 3 shows the effect of the introduction of a self-induction, the conditions of the discharge remaining otherwise the same as in photograph No. 1.

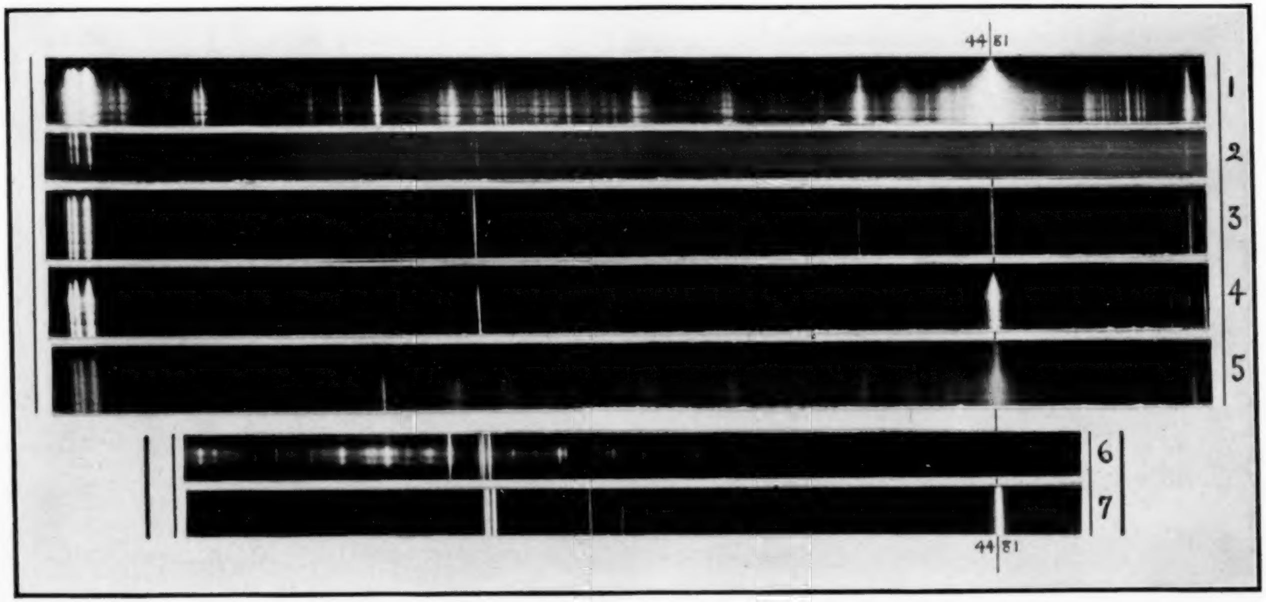
In spectrum No. 5 a stronger alternating current and a capacity four times as great as in No. 1 were employed; but the photograph is feeble from over-exposure. On the contrary, in No. 4 the coil was excited by a feeble continuous current and the capacity in the circuit was reduced to a small jar.

The two spectra placed below were taken some years ago with another spectroscope. They are of interest in showing the great variation of intensity which the line at λ 4481 may undergo without assuming its normal diffused character, as in No. 1. The line in spectrum No. 4 appears to be intermediate in character between that of the line in No. 7 and that in No. 1.

We have still before us the task of the interpretation of the differences in the mode of production of the electric spark, especially as to greater or less suddenness, so as to enable us to connect them with definite conditions of temperature and density of those stellar atmospheres in which the magnesium present absorbs radiations similar in character to those photographed in the laboratory.

London,
January 1, 1903.

PLATE XIV.



SPARK SPECTRA OF MAGNESIUM.

ON THE FLAME SPECTRUM OF RADIUM.

By C. RUNGE and J. PRECHT.

DR. GIESEL has recently found that very small quantities of sufficiently pure radium bromide give a characteristic color to the flame of a Bunsen burner.[1] He describes the spectrum on the whole as consisting of two red bands and a bright blue line. In the violet he saw some other less prominent lines.

At the request of Dr. Giesel we have investigated the spectrum more closely, for which purpose he was kind enough to supply us with 13 milligrams of radium bromide of his own making.

To fix the substance on the platinum wire we proceeded in the same way as described by one of us in his paper on the spark spectrum of radium.[2] The platinum wire was doubled up, twisted into a loop, and heated by an electric current. If a fragment of the salt is touched with the glowing wire, it sticks to it, melts, and forms a small drop. When the current is cut off, the drop hardens and begins to give out a bright phosphorescent light. If the wire is again heated, the light ceases at once, but is renewed as soon as the heating is stopped.[3]

The wire carrying a small fragment of the substance was brought into the hottest part of the Bunsen flame, which was placed before the slit of a spectroscope. The spectroscope consisted of a Rowland concave grating of one meter radius. It was mounted in the way described by one of us.[4] If the wire is electrically heated while the substance is in the flame, the intensity of the spectrum is increased, but the substance is consumed more rapidly.

The wave-lengths were read off on a paper scale. The light

[1] F. GIESEL, *Physikal. Zeitschrift*, **3**, 578, 1901–2 and *Ber. d. deutsch. chem. Gesellschaft*, **35**, 3608, 1902.

[2] C. RUNGE, "On the Spectrum of Radium," ASTROPHYSICAL JOURNAL, **12**, 1.

[3] This phenomenon has already been described by Giesel.

[4] *Wied. Ann.*, **61**, 644, 1897.

is not sufficient for a photograph if only small quantities of the salt are at the disposal of the observer.

When the substance is first brought into the flame, there are some lines to be seen, which rapidly lose their intensity, so much so that one must be quick to determine their wave-lengths before they disappear. The blue line at λ 4826 is the last to lose its intensity. It seems to possess the same character as the barium line λ 5536, the strontium line λ4607, and the calcium line λ4226, which are also most prominent when these substances are brought into the Bunsen flame.

In the following table we have noted all we have seen. We believe all the observed lines to be due to radium except the barium line λ5536 and the two sodium lines. The wave-lengths

Wave-Lengths	Remarks	
4405	Weak, diffuse	
4500	Weak, diffuse, observed only once	
4592	Diffuse	
4680	Weak, observed only once, perhaps identical with the strong line of the spark spectrum at λ4682.3	
4718		
4750		
4826	Strong, sharp line, very likely identical with the line at λ4826.1 in the spark spectrum, as Giesel presumes	
509–513	Weak band, observed only once	
5210	Observed only once	
5360	Observed only once	
5535	Impurity, barium λ 5535.7	
5655	Observed only once	
5685	Observed only once	
5890	Impurity, sodium	
5896	Impurity, sodium	
590–605	Weak band	
613–633	Strong band	
6210		These lines are seen in the band, but apparently do not form the band
6216	Observed only once	
6228	Observed only once	
6247		
6250	Observed only once	
6260	Observed only once	
6269		
6285		
6329	Strong	
6349	Strong	
653–670	Strong band	
6653	Strong line; there is a minimum of intensity of the band, in the neighborhood of the line	
6861	Uncertain, observed only once	

of the lines that have been observed clearly and repeatedly we believe to be correct to one or two Ångström units. With those lines, however, which have been observed only once, the error may amount to several Ångström units. The bands have no definite limits. When an ample amount of the salt is brought into the flame, they may appear much broader than we have seen them. It may be that the bands are due to the spectrum of the compound. This might be tested by observing the flame spectrum of the chloride.

HANNOVER,
January 1903.

FIVE STARS WHOSE RADIAL VELOCITIES VARY.

By Edwin B. Frost and Walter S. Adams.

In the course of the observations we have been making with the Bruce spectrograph during the past year on stars having spectra of the *Orion* type, the following four stars of this class have been found to vary in respect to their motions in the line of sight, in addition to the three previously announced in this journal (*η Orionis*, *o Persei*, *β Cephei*).[1]

The determinations of velocity so far made are given below. The G. M. T. of the observations is not now communicated, as we do not deem the data at present available to be sufficient for the accurate determination of the orbits of these stars. The number of star lines measured is given for each plate in the fifth column.

δ CETI ($\alpha=4^h\ 31^m$; $\delta=-0°\ 6'$; Mag.=4.1)

Plate	Date	Taken by	Velocity	No. of Lines	Measured by
A 291	1901, Nov. 1	F.	+ 8 km	15	A.
B 227	Nov. 13	F.	+12	10	F.
A 295	Dec. 19	A.	+16	7	A.
A 298	1902, Jan. 4	A.	+ 6	7	A.
B 383	Aug. 7	F.	+13	8	A.
B 388	Aug. 11	A.	+ 6	9	A.
B 400	Aug. 27	A.	+ 9	9	A.
B 404	Sept. 3	A.	+12	8	A.
A 376	Sept. 6	A.	+12	10	A.
A 381	Sept. 7	F.	+ 6	10	A.
B 432	Oct. 29	A.	+ 9	8	A.

The spectrum is of Miss Maury's Class IV*a* and has numerous oxygen lines and the usual ones of helium, silicon, hydrogen, and magnesium. The lines are fairly sharp, so that the spectrum is relatively well measurable. The above range of variation in velocity is not large, from +6 to +16 km per sec., but we cannot question its reality.

[1] Astrophysical Journal, 15, 214, 340, 1902; 17, 68, 1903.

ν ERIDANI ($\alpha = 4^h\ 31^m$; $\delta = -3°\ 33'$; Mag. $= 4.1$)

Plate	Date	Taken by	Velocity	No. of Lines	Measured by
B 242	1901, Nov. 20	A.	+12 km	5	A.
B 438	1902, Oct. 30	F.	{+28 +26	10 12	F. A. }
B 443	Oct. 31	A.	+18	7	A.
B 447	Nov. 6	A.	+ 3	8	A.
B 472	Dec. 18	F.	+12	8	F.
B 494	1903, Feb. 5	A.	+25	7	F.

The spectrum resembles that of δ *Ceti*, but the lines are more difficult to measure. The range so far observed is 24 km.

π^5 *ORIONIS* ($\alpha = 4^h\ 49^m$; $\delta = +2°\ 17'$; Mag. $= 3.9$)

Plate	Date	Taken by	Velocity	Number of Lines	Measured by
A 332	1902, March 4	A.	+ 1	3	A.
B 469	Dec. 17	A.	+ 58	5	A.
B 475	Dec. 31	A.	{+ 70 + 72	5 6	A. F. }
B 480	1903, Jan. 1	Ellerman	+ 32	4	A.
A 384	Jan. 16	A.	+ 7	6	A.
B 488	Jan. 21	F.	{− 35 − 32	4 4	A. F. }
A 390	Jan. 22	A.	+ 73	4	A.

The spectrum of this star is not well adapted for accurate measurement, the lines being broad and very diffuse. A few oxygen lines are present, but they are faint and difficult to set upon. The period is evidently short.

ζ *TAURI.*

The spectrum of the star is peculiar, as has been noted by Lockyer and probably by others. The conspicuous feature in the region of spectrum covered by our plates is the remarkable sharpness and intensity of $H\gamma$. Settings can be made upon this line with such accordance that the radial velocity determined from this line alone is probably not much less reliable than that based on several lines in the case of most other stars of the *Orion* type. Other lines are faintly discernible in the spectrum, chiefly the enhanced lines of titanium and iron. These lines are in general so faint and broad, however, that they have commonly not been employed in the determination of the star's motion.

ζ *TAURI* ($\alpha = 5^h\ 32^m$; $\delta = +21°\ 5'$; Mag. = 3.0).

Plate	Date	Taken by	Velocity	Number of Lines	Measured by
B 219	1901, Nov. 8	A.	+17	1	A.
			+16	1	F.
A 317	1902, Feb. 12	A.	+23	1	A.
			+24	1	F.
B 332	April 23	F.	+15	1	A.
			+11	1	F.
B 410	Sept. 13	A.	+18	6	A.
B 425	Oct. 15	A.	+34	1	A.
B 440	Oct. 30	F.	+31	1	A.
			+28	2	F.
B 452	Nov. 6	A.	+19	1	A.
			+21	1	F.
B 462	Nov. 19	A.	+14	1	A.
B 470	Dec. 17	A.	+7	1	A.
B 473	Dec 18	F.	+9	1	F.
B 476	Dec. 31	A.	+4	1	A.
B 482	1903, Jan. 8	A.	+2	1	A.
B 485	Jan. 9	A.	+5	1	A.
A 386	Jan. 16	A.	+4	1	A.

The period is probably rather long. Plate B 452 of the above series is underexposed, and the value derived from it is somewhat uncertain.

η *VIRGINIS* ($\alpha = 12^h\ 15^m$; $\delta = -0°\ 7'$; Mag. = 4.1).

This star has a composite spectrum, both components belonging to Vogel's type I*a* 2, or Miss Maury's VIII*a*. On account of the weakness of the lines of the fainter component the discussion of the star's motion will be based mainly upon the absolute velocity of the brighter component. The binary character of the star was established by the first plate, taken and measured by A.

Plate	Date	Taken by	Brighter Component		Fainter Component		Measured by
			Velocity	No. of Lines	Velocity	No. of Lines	
A 388	1903, Jan. 16	A.	−31.5	14	+42	4	A.
B 493	Feb. 4	F.	+0.7	16	+60	4	F.
			+0.2	15	+63	6	A.
A 399	Feb. 5	A.	+3.4	16	..	..	A.

NOTE ADDED FEBRUARY 18, 1903.

When it was decided to publish at this time the observations of the above stars, we expected also to include the *Orion* type star π^4 *Orionis*, the first two plates of which indicated a range of variation of about 15 km. It was, however, withheld until we should obtain additional plates, as the third and fourth plates gave results nearly like that of the second. We see today in the *Publications of the Astronomical Society of the Pacific* (**15**, 20, 1903) a notice by Dr. H. M. Reese, stating that spectrograms of this star, taken with the Mills spectrograph of the Lick Observatory, on October 6, 1902, and January 4 and 12, 1903, yielded values respectively of $+43$, ± 0, and $+6$ km. per second.

There is accordingly no reason for longer withholding our observations, which are as follows:

π^4 *ORIONIS* ($\alpha = 4^h\,46^m$; $\delta = +5°\,26'$; Mag. 4.0)

Plate	Date	Taken by	Velocity	Number of Lines	Measured by
A 332	1902, March 4	A.	+13 km	8	A.
B 466	Nov. 27	F.	− 2	5	F.
			− 1	7	A.
B 468	Dec. 17	A.	+ 1	6	F.
A 389	1903, Jan. 22	F.	+ 3	7	F.
			+ 0	6	A.

YERKES OBSERVATORY,
February 8, 1903.

SECOND NOTE ON THE SPARK SPECTRUM OF IRON IN LIQUIDS AND COMPRESSED GASES.

By George E. Hale and Norton A. Kent.

In a note published a year ago[1] it was shown that the position and the reversal phenomena of the lines in the spectrum of a spark between iron poles in liquids depend upon a variety of circumstances, chief among which are the electrical constants of the circuit and the nature of the liquid. It was also stated that certain lines in the blue part of the spectrum of iron had been photographed in air at pressures ranging from 1 to 20 atmospheres, and that the shifts of the lines were (approximately) directly proportional to the pressure, thus confirming and extending the results previously found by Humphreys and Mohler for the low potential discharge in air at pressures up to 14½ atmospheres. It was at the same time remarked that the lines thus investigated in air showed none of the peculiar phenomena of reversal which had been observed in the case of the high potential discharge in liquids.

Since this first note was written, the investigation of the spectrum of iron in liquids has been completed. The reversal phenomena previously studied with low dispersion have been photographed with a concave grating of ten feet (3.05 m) radius, on a scale sufficiently great to permit the displacements of the best lines to be measured with errors not often exceeding 0.01 tenth-meter. In the note referred to above it was stated that the preliminary experiments on the effect of self-induction in the discharge circuit were inconclusive. With a coil afterwards constructed, in which the self-induction can be varied from 0.000042 to 0.000426 henry in five steps, change of self-induction has been found to produce more marked effects than change of capacity, length of spark, diameter of terminals, physical or chemical properties of the liquid, or any other variable previously shown to influence the reversal phenomena. Accordingly,

[1] George E. Hale, "Note on the Spark Spectrum of Iron in Liquids and in Air at High Pressures," Astrophysical Journal, **15**, 132, 1902.

changes of self-induction in the discharge circuit have been employed to produce a series of spectra, which pass by gradual degrees from a spectrum consisting almost wholly of bright lines (highest self-induction), resembling that of the iron spark in air at atmospheric pressure, to a spectrum consisting for the most part of dark lines (no self-induction) in the region $\lambda 3550$–$\lambda 4500$. Such a series of spectra, if photographed with low dispersion, would closely resemble the spectra reproduced in Plate XI of the note referred to above. A careful series of measures of the lines has shown that as the self-induction is decreased the bright lines move gradually toward the red, while the absorption lines, which begin to appear in the earliest stages, may at first have some apparent shift[1] toward the violet, though later they may be displaced one or two hundredths of a tenth-meter toward the red, rarely more. Full details of the measurements will be given in a paper which will appear soon in the *Publications of the Yerkes Observatory*.

In his paper "On the Interpretation of the Typical Spectrum of the New Stars,"[2] Professor Wilsing describes his experiments on the spectrum of high potential discharges between metallic poles in water, and bases an explanation of the characteristic pairs of bright and dark lines in the spectra of temporary stars upon the phenomena he observed. Wilsing concludes that the pressure resulting from the spark discharge in liquids amounts to several hundred atmospheres. As our own investigations led us to the belief that the pressure, at least in the case of our iron spark, was in reality much lower, we thought it desirable to extend our earlier work on spark spectra in air at high pressures to the more refrangible region studied in the case of liquids. As the apparatus used for such work a year ago proved inadequate, new apparatus has been constructed in the Observatory instrument shop, and with this we have had no difficulty in photographing the spectrum of the iron spark in gases at pressures ranging from 1 to 53 atmospheres. The present note contains a preliminary account of the results hitherto obtained, but many

[1] Doubtless due in part to the displacement of the lines of the comparison spectrum, caused by pressure in the condensed spark.

[2] ASTROPHYSICAL JOURNAL, **10**, 113, 1899.

details must be reserved for the more complete paper which is to appear in the Observatory *Publications*.

In the recent experiments the spark between iron terminals has been observed through a glass window, in the side of a steel chamber provided with a pressure gauge, and connected by means of a heavy copper tube with a steel reservoir containing compressed air or liquid carbon dioxide. The terminals enter the steel pressure chamber through long cylinders of ebonite, and special precautions have been taken to insure the most perfect insulation. The spark is produced by means of a transformer giving about 15,000 volts on open circuit, ordinarily used with a condenser of 0.0066 microfarad capacity, connected with the secondary terminals. In order to insure successful operation of the spark at high pressures, an air break is essential in the discharge circuit. The photographs of spectra reproduced in Plate XV were made in the first order of a concave grating of ten feet radius, ruled with 14,438 lines to the inch (5684 to the cm).

As stated above, the spectra photographed a year ago at high pressures did not extend into the ultra-violet. In a previous note[4] it has been shown that the reversals of the lines in the case of spark spectra in liquids first appear in the ultra-violet, and advance gradually toward the less refrangible region as the conditions for reversal become more favorable. A similar result was shown many years ago by Liveing and Dewar to obtain in the case of arc reversals. It is for this reason that the phenomena described in the present note were not encountered by us in our earlier work at lower pressures on the less refrangible region of the spectrum of the iron spark in air. Nevertheless, we were somewhat surprised to find, in our first test of the new apparatus, that at an air pressure of 14 atmospheres the reversal phenomena in the region $\lambda 3550$–$\lambda 4500$ closely resembled those we were accustomed to photograph in the case of the iron spark in water with but little self-induction in the discharge circuit. It was at once evident that a series of photographs made at different air

[4] GEORGE E. HALE, "Selective Absorption as a Function of Wave-Length," ASTROPHYSICAL JOURNAL, **15**, 227, 1902.

pressures would resemble the series previously obtained with the spark in water by varying the self-induction. Photographs have now been made at pressures ranging from 1 to 53 atmospheres, portions of which are reproduced in Plate XV. In the temporary absence of compressed air at pressures greater than 14 atmospheres, liquid carbon dioxide was used to give higher pressures. It has since been found, however, that at a given pressure the reversal phenomena are more marked in an atmosphere of air than in one of carbon dioxide. This result will render necessary an investigation of the effect of other gases. It has also been found that changes of self-induction in the condenser circuit affect the reversal phenomena in the same sense as in the case of the spark in water, but over a much smaller range.

An examination of Plate XV will show that the phenomena closely resemble those previously described in the case of liquids. At an air pressure of three atmospheres the lines are for the most part bright, though a few cases of reversal appear. It will be noticed that a few of the lines have at this stage increased very markedly in relative intensity. Such lines, as the subsequent photographs show, pass through a maximum of intensity and afterwards reverse. At a pressure of 7 atmospheres the bright lines are broader and more diffuse, and the absorption lines are becoming prominent. It is evident from inspection that many of the reversals are not symmetrical, the bright line being relatively displaced toward the red. At a pressure of 14 atmospheres the dark lines have become very conspicuous, and many of the bright lines have almost disappeared. Some evidences of a continuous spectrum begin to appear at this pressure. At 27 atmospheres (in carbon dioxide) the continuous spectrum is quite conspicuous. The absorption lines are strong and the bright lines are much fainter than before. At 53 atmospheres (also in carbon dioxide) the bright lines have practically disappeared and the dark lines remain, broad and diffuse, on a background of bright continuous spectrum. At this pressure most of those lines whose intensity is increased at moderate pressures are reversed, and the relative intensities of all the dark lines closely resemble those of the bright lines in the spectrum

of the iron spark at atmospheric pressure. Even at the highest pressures very few lines less refrangible than λ 4415 are reversed. In the region λ 4800–λ 4900 there are several strong lines unsymmetrically broadened in carbon dioxide at 53 atmospheres, and similarly affected in water. It seems to be true that the order in which various lines reverse is not precisely the same in gases as in liquids.

A careful study of the shifts of the bright and dark lines at these various pressures has brought out certain facts, which are of interest in connection with the results obtained by Humphreys and Mohler on the low potential discharge in air at high pressures, and our own results on the spectra of high potential discharges in liquids. The following tables contain measures of a few lines which may be regarded as fairly typical. Table I gives the shifts of certain lines in the spectrum of a spark between iron poles in water, resulting from changes of self-induction in the discharge circuit. In this series the length of the water spark was 0.3 mm; air spark 10 mm; diameter of terminals (flat ends) in water 2.3 mm; diameter of terminals (rounded ends) in air 4 mm; metal in both cases Bessemer steel; electric fan used to prevent arcing at air gap; distance of terminals below surface of water 56 mm; 15,000 volt transformer; capacity in discharge circuit 0.0066 microfarad. The shifts of the lines toward the red are expressed in tenth-meters, and are positive unless otherwise indicated. The types of the lines, as indicated below, are given in parentheses.

(1) = narrow emission line.

(2) = broad symmetrical emission line.

(3) = broad emission line, diffuse toward red.

(4) = symmetrical absorption line.

(5) = absorption line superposed symmetrically on broad emission line.

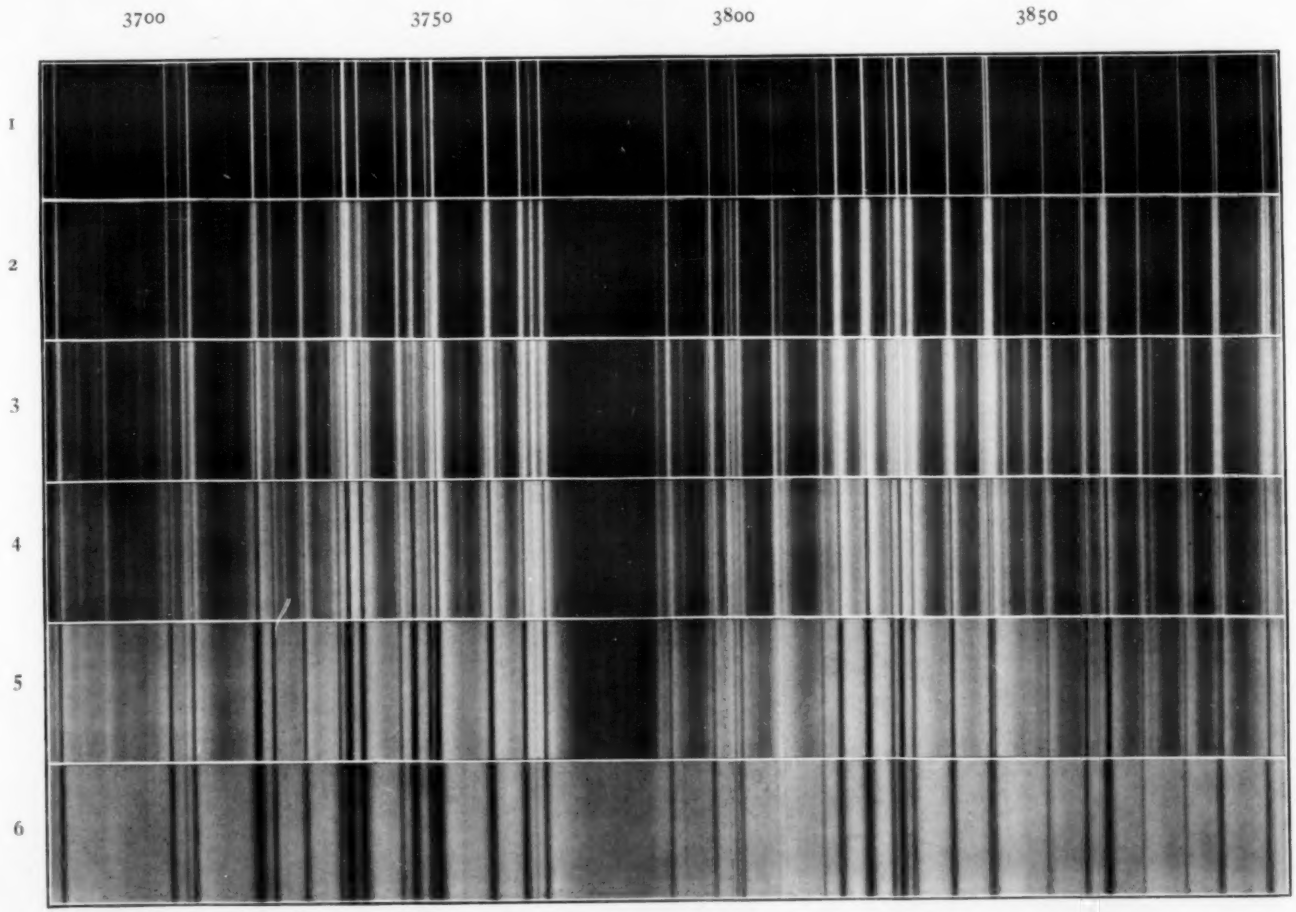
(6) = absorption line superposed on broad emission line, which is strongest on red side.

(7) = similar to (6), but with violet component of bright line lacking.

E signifies that the measured shift refers to an emission line, *A* that it refers to an absorption line. In lines of types (6) and (7) both the emission and absorption components may be measured. In such a case the measurement of the red component (the violet component is not measured) of the emission line does

PLATE XV.

PLATE XV.



Photographs of the Spark Spectrum of Iron in Compressed Gases.

1. In air at atmospheric pressure.
2. In air at 3 atmospheres.
3. In air at 7 atmospheres.
4. In air at 14 atmospheres.
5. In CO_2 at 27 atmospheres.
6. In CO_2 at 53 atmospheres.

not give the true shift of the line, since the setting is made at the center of that part of the line which lies on the red side of the absorption line. Such measures are given here incidentally only because of their previous use by Wilsing.

TABLE I.

IRON SPARK IN WATER.

Shifts corresponding to changes of self-induction in discharge circuit.

Plate Number		659	651	650	658	666	665
Self-Induction Henries		0.000426	0.000310	0.000161	0.000080	0.000042	0.0
Wave-Length	Intensity						
		t. m.	*t. m.*	*t. m.*	*t. m.*	*t. m.*	*t. m.*
3606.85	6	(2) 0.01 *E*	(2) 0.02 *E*	(2) 0.04 *E*	(2) 0.08 *E*	(2) 0.14 *E*	(7)-0.01 *A* .28 *E*
3765.70	5	(2) .02 *E*	(2) .03 *E*	(2) .01 *E*	(2) .09 *E*	(2) .10 *E*	(3) .21 *E*
3815.99	9	(3) .05 *E*	(3) .07 *E*	(3) .12 *E*	(6) .00 *A*	(6) .01 *A*	(6) .03 *A*
3827.98	9	(3) .02 *E*	(3) .05 *E*	(2) .09 *E*	(7)- .01 *A* .35 *E*	(6) .00 *A*	(4) .02 *A*
4063.75	10	(2) .03 *E*	(2) .02 *E*	(2) .04 *E*	(6) .06 *A*	(6) .02 *A*	(4) .02 *A*
4294.32	6	(2) .02 *E*	(2) .02 *E*	....	(2) .02 *E*	(2) .06 *E*	....
4308.06	10	(2) .02 *E*	(3) .05 *E*	(2) .06 *E*	(6)- .05 *A*	(6)- .01 *A*	(6) .02 *A*
4325.94	10	(3) .02 *E*	(3) .04 *E*	(2) .03 *E*	(6)- .01 *A*	(6)- .03 *A*	(6) .01 *A*

TABLE II.

IRON SPARK IN AIR AND CO_2.

Shifts corresponding to pressures ranging from 3 to 53 atmospheres.

Plate Number		842	839	833	853	852
Pressure in Atmospheres		Air 3	Air 7	Air 14	CO_2 27	CO_2 53
Wave-Length	Intensity					
		t. m.	*t. m.*	*t. m.*	*t. m.*	*t. m.*
3606.85	6	(2) 0.06 *E*	(2) 0.14 *E*	(7) 0.05 *A* .39 *E*	(4) 0.05 *A*	(4) 0.12 *A*
3765.70	5	(2) .09 *E*	(2) .15 *E*	(7) .05 *A* .33 *E*	(4) .07 *A*	(4) .12 *A*
3815.99	9	(6) .04 *A*	(6) .05 *A*	(6) .08 *A*	(4) .08 *A*	(4) .14 *A*
3827.98	9	(2) .11 *E*	(6) .06 *A*	(6) .10 *A*	(6) .07 *A*	(4) .20 *A*
4063.75	10	(2) .07 *E*	(5) .06 *A*	(5) .07 *A*	(5) .10 *A*	(5) .18 *A*
4294.32	6	(2) .04 *E*	(2) .09 *E*	(2) .16 *E*	(2) .21 *E*	....
4308.06	10	(2) .09 *E*	(5) .08 *A*	(5) .09 *A*	(5) .12 *A*	(5) .18 *A*
4325.94	10	(2) .07 *E*	(5) .04 *A*	(5) .10 *A*	(5) .09 *A*	(5) .22 *A*

Table II contains the shifts of the same lines as observed at pressures of 3, 7, and 14 atmospheres in air, and at 27 and 53

atmospheres in carbon dioxide. The constants of the circuit were the same as in the case of water, except that the external air gap was 5 mm long.

As Humphreys and Mohler have shown, the reversals in the case of the arc in air at high pressures are generally symmetrical, the absorption lines being displaced toward the red equally with the emission lines. In the case of the spark in water we have found, as the table shows, that the absorption lines are but little displaced toward the red, though they sometimes have a small apparent displacement toward the violet. The emission lines, however, are considerably displaced toward the red, and this displacement increases as the self-induction in the discharge circuit decreases. A knowledge of the true displacement of the bright lines is obtained from a study of those lines which do not reverse before the last stages of the process are reached. Thus the errors resulting from the presence of the absorption lines are avoided. At moderate pressures the absorption lines are displaced less than the bright lines, though their displacements are considerably greater than in the spectrum of the spark in liquids.

These are only a few of the lines which we have already measured, but much additional material will be required before the results can be discussed satisfactorily. It may be noted, however, that the relative shifts of several lines in air and water indicate that the pressure produced by our iron spark in water (no self-induction in the discharge circuit) is much less than Wilsing's value of several hundred atmospheres. An extension to high pressures of the law connecting pressure and displacement will be possible as soon as a complete series of spectra has been obtained in a single gas. But on account of the lack of agreement among the shifts obtained for different lines, and the possibility that in the high potential discharge the pressure of the gas is not the sole controlling factor, it is hoped that the present work may be supplemented by an investigation of the discharge at potentials ranging from 110 volts to 15,000 volts.

YERKES OBSERVATORY,
February 11, 1903.

Minor Contributions and Notes.

REPLY TO E. VON OPPOLZER'S REMARKS ON BIGELOW'S "ECLIPSE METEOROLOGY."[1]

On p. 738 of *Astronomy and Astro-Physics*, **12**, 1893, E. von Oppolzer writes: "Assuming that the solar atmosphere is made up of hydrogen, the diminution of temperature per second $[1'' = 720860\,\mathrm{m}]$ is $\theta = 13740°$. At a distance of one second above the photosphere, we have, therefore, a prevailing temperature 14000° lower than on the surface of the photosphere." Inasmuch as the prevailing temperature of the photosphere is by general consent in the neighborhood of 7000° C., this statement is so far in error as to discredit the theory upon which the result is based. His description in the note mentioned of the work summarized in my *Eclipse Meteorology*, pp. 77–83, is hopelessly in error, and the reader is referred to my original text. There should be no difference of opinion about the deduction of the formulæ employed, as they follow the lines laid down in my reports, *International Cloud Observations*, 1898, pp. 487–490, or *Barometry of the United States*, 1902, chap. 2, which is common meteorology. Since the height of a homogeneous atmosphere on the Sun has a fundamental bearing in the studies of the pressures and temperatures of the solar atmosphere, it may be proper to deduce this constant again, and to explain the terms that enter into it. A homogeneous atmosphere is one which has throughout its length a constant mass per unit volume, this mass being that of the lowest cubic volume of the gas composing the atmosphere itself, and the total pressure being that of the actual atmosphere. In the case of the Earth this pressure is, in units of weight,

$$p = \sigma_m B_n = 13595.8 \times 0.760 \quad \text{in kilograms/meter}^2,$$
$$= \sigma_o l \quad = 1.29305 \times 7991.04,$$

where σ_m, σ_o are the weights of the unit volume of mercury and air, and B_n, l the heights of the homogeneous columns, respectively.

Similarly the terms may be computed for a hydrogen atmosphere on the Sun, as follows: To deduce the mass of the unit volume at the

[1] Astrophysical Journal, **16**, 334, 1902.

Sun from the terrestrial constants, we must take the pressure and temperature of the Sun's photosphere. Gravity is $g = 27.6\, g_o$, and the temperature I have assumed to be $7535^\circ = 27.6 \times 273$, or $T = 27.6\, T_o$. There are two reasons for using $T = 7535^\circ$C.: (1) The computed temperature of the solar surface hovers around 7000°C., while the figure 7535 "serves to systematize the solar and terrestrial constants, for hydrogen and air respectively, in a single numerical scheme," as given on p. 77. (2) The observed ordinates of the normal solar spectrum, from Professor Langley's results, in the region of the wave-lengths $1.5\,\mu$ to $1.7\,\mu$ are longer than correspond to radiation temperatures of less than $T = 7535^\circ$. There seem to be certain waves which pass through both the solar and terrestrial envelopes without depletion, and which will not match any lower temperature curves. This is shown in the *Monthly Weather Reviews* December 1902. Taking the usual notation, Earth $(P_o, T_o, \sigma_o, g_o)$, Sun (P, T, σ, g), we have

$$P_o = g_o \sigma_m B_n \qquad \text{for the Earth,}$$

and

$$P = 27.6\, g_o \sigma_m B_n \quad \text{for the Sun.}$$

Since the density varies by the formula,

$$\sigma = \sigma_o \frac{P}{P_o} \frac{T_o}{T} = \sigma_o \frac{27.6\, P_o}{P_o} \cdot \frac{T_o}{27.6\, T_o} = \sigma_o \ ,$$

it is inferred that we can use the terrestrial densities on the Sun for the temperature $T = 7535^\circ$, and thus unify the two systems in one numerical scheme. It is evident that I have not omitted the gravity factor in $g = n \, . \, g_o$, as von Oppolzer stated in his notes.

For the homogeneous atmosphere of hydrogen we have, by the thermodynamic and the dynamic formulæ, respectively,

$$l = RT = 420.552 \times 7535 = 3168860 \text{ m} \ ,$$

and

$$l = \frac{\sigma_m B_n \times 27.6}{\sigma_n} = \frac{13595.8 \times .760 \times 27.6}{0.089996} = 3168860 \text{ m} \ .$$

Hence the barometric constant for common logarithms is $K = \dfrac{l}{M}$ $= 7296570$, as used in my formulæ.

On p. 77 there are numerous checks by cross-computations among the formulæ, and the numbers are consistent with one another. From this point onward the discussion follows well-known lines, and they need not be indicated more fully. My results vary somewhat in detail from Fényi's, but generally agree closely enough to say that we have reached similar conclusions by different methods.

It should be noted that, if a pressure of five atmospheres be assumed

for the upper strata of the photosphere, the computed diminution of pressure outward seems to be in harmony with the observed extent of the inner corona. Also, from these data one may compute the pressures on the inner strata below the surface of the photosphere, assuming the structure to be arranged practically in adiabatic layers, and thus construct an approximate solar meteorological system. It is very desirable to determine whether this use of the temperature $T = 7535°$ is not permissible from the observational data. It would make the solar constant at the Earth about 4.0 gram-calories per minute, and there are reasons for thinking it is higher than 3.0 gram-calories. Indeed, there are several directions in which these constants can be utilized, unless my suggestion meets with well considered objections.

FRANK H. BIGELOW.

WEATHER BUREAU,
January 5, 1903.

CORRECTION.

IN the December number, p. 337, of the ASTROPHYSICAL JOURNAL, the visual rediscovery of *Eros* last August at the Chamberlin Observatory of the University of Denver was attributed to me. The rediscovery was really made by Dr. Charles J. Ling, to whom the credit is due.

HERBERT A. HOWE.

ERRATA.

ASTROPHYSICAL JOURNAL, Vol. **16**, November 1902:

Page 189, third line from foot, *for* 63300, *read* 63300 π.
190, fourth line from foot, *for* r, *read* z.
190, fourth line from foot, *for* γ, *read* γ^2.
190, last line, *for* (m^2-n^2), *read* $(m^2-n^2)^2$.
191, sixth line from top, *for* zcn, *read* $2cn$.

Vol. **16**, December 1902:

In equation (23) the first term within the bracket $= D$, should be written $\frac{3}{2}\rho$ instead of ρ and the second term should be $\frac{\rho^2}{u}$ instead of ρ.

In the three equations following (31) *for* $\theta_{max} = 2° \ 20'$ and $\theta_{max} = 6°$, *read* $\theta_{max} = 0.°22$ and $\theta_{max} = 0.°6$ respectively.

Vol. **17**, No. 1, January 1903:

Plate I, exchange $-15'$ and $+15'$.
III, *for* Light Curve, *read* Velocity Curve.

NOTICE.

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

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